University of Debrecen Faculty of Science and Technology Institute of Mathematics

# MATHEMATICS BSC PROGRAM

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# **DEAN'S WELCOME**

Welcome to the Faculty of Science and Technology!

This is an exciting time for you, and I encourage you to take advantage of all that the Faculty of Science and Technology UD offers you during your bachelor's or master's studies. I hope that your time here will be both academically productive and personally rewarding

Being a regional centre for research, development and innovation, our Faculty has always regarded training highly qualified professionals as a priority. Since the establishment of the Faculty in 1949, we have traditionally been teaching and working in all aspects of Science and have been preparing students for the challenges of teaching. Our internationally renowned research teams guarantee that all students gain a high quality of expertise and knowledge. Students can also take part in research and development work, guided by professors with vast international experience.

While proud of our traditions, we seek continuous improvement, keeping in tune with the challenges of the modern age. To meet the demand of the job market for professionals, we offer engineering courses with a strong scientific basis, thus expanding our training spectrum in the field of technology. Based on the fruitful collaboration with our industrial partners, recently, we successfully introduced dual-track training programmes in our constantly evolving engineering courses.

We are committed to providing our students with valuable knowledge and professional work experience, so that they can enter the job market with competitive degrees. To ensure this, we maintain a close relationship with the most important national and international companies. The basis for our network of industrial relationships are in our off-site departments at various different companies, through which market participants - future employers - are also included in the development and training of our students.

Prof. Dr. Ferenc Kun Dean

# **UNIVERSITY OF DEBRECEN**

Date of foundation: 1912 Hungarian Royal University of Sciences, 2000 University of Debrecen

Legal predecessors: Debrecen University of Agricultural Sciences; Debrecen Medical University; Wargha István College of Education, Hajdúböszörmény; Kossuth Lajos University of Arts and Sciences

Legal status of the University of Debrecen: state university

Founder of the University of Debrecen: Hungarian State Parliament

Supervisory body of the University of Debrecen: Ministry of Education

#### Number of Faculties at the University of Debrecen: 13

Faculty of Agricultural and Food Sciences and Environmental Management

Faculty of Child and Special Needs Education

Faculty of Dentistry

Faculty of Economics and Business

Faculty of Engineering

Faculty of Health

Faculty of Humanities

Faculty of Informatics

Faculty of Law

Faculty of Medicine

Faculty of Music

Faculty of Pharmacy

Faculty of Science and Technology

# Number of students at the University of Debrecen: 30,899

# Full time teachers of the University of Debrecen: 1,597

210 full university professors and 1,262 lecturers with a PhD.

# FACULTY OF SCIENCE AND TECHNOLOGY

The Faculty of Science and Technology is currently one of the largest faculties of the University of Debrecen with about 2,500 students and more than 200 staff members. The Faculty has got 6 institutes: Institute of Biology and Ecology, Institute of Biotechnology, Institute of Chemistry, Institute of Earth Sciences, Institute of Physics and Institute of Mathematics. The Faculty has a very wide scope of education dominated by science and technology (12 Bachelor programs and 14 Master programs), additionally it has a significant variety of teachers' training programs. Our teaching activities are based on a strong academic and industrial background, where highly qualified teachers with a scientific degree involve student in research and development projects as part of their curriculum. We are proud of our scientific excellence and of the application-oriented teaching programs with a strong industrial support. The number of international students of our faculty is continuously growing (currently ~760 students). The attractiveness of our education is indicated by the popularity of the Faculty in terms of incoming Erasmus students, as well.

### THE ORGANIZATIONAL STRUCTURE OF THE FACULTY

Dean: Prof. Dr. Ferenc Kun, Full Professor E-mail: <u>ttkdekan@science.unideb.hu</u>

Vice Dean for Educational Affairs: Prof. Dr. Gábor Kozma, Full Professor E-mail: <u>kozma.gabor@science.unideb.hu</u>

Vice Dean for Scientific Affairs: Prof. Dr. Sándor Kéki, Full Professor E-mail: <u>keki.sandor@science.unideb.hu</u>

Consultant on External Relationships: Prof. Dr. Attila Bérczes, Full Professor E-mail: <u>berczesa@science.unideb.hu</u>

Consultant on Talent Management Programme: Prof. dr. Tibor Magura, Full Professor E-mail: <u>magura.tibor@science.unideb.hu</u>

Dean's Office Head of Dean's Office: Mrs. Katalin Kozma-Tóth E-mail: <u>toth.katalin@science.unideb.hu</u>

English Program Officer: Mrs. Alexandra Csatáry Address: 4032 Egyetem tér 1., Chemistry Building, A/101, E-mail: <u>acsatary@science.unideb.hu</u>

# **DEPARTMENTS OF INSTITUTE OF MATHEMATICS**

**Department of Algebra and Number Theory** (home page: https://math.unideb.hu/en/introduction-department-algebra-and-number-theory)

Name	Position	E-mail	room
Prof. Dr. Attila	University Professor,	berczesa@science.unideb.hu	M415
Bérczes	Head of Department		
Prof. Dr. István Gaál	University Professor	gaal.istvan@unideb.hu	M419
Prof. Dr. Lajos	University Professor	hajdul@science.unideb.hu	M416
Hajdu			
Prof. Dr. Ákos Pintér	University Professor	apinter@science.unideb.hu	M417
Prof. Dr. Szabolcs	University Professor	tengely@science.unideb.hu	M415
Tengely			
Dr. Gábor Nyul	Associate Professor	gnyul@science.unideb.hu	M405
Dr. István Pink	Associate Professor	pinki@science.unideb.hu	M405
Dr. András Bazsó	Assistant Professor	bazsoa@science.unideb.hu	M407
Dr. Nóra Györkös-	Assistant Professor	nvarga@science.unideb.hu	M417
Varga			
Dr. Gabriella Rácz	Assistant Professor	racz.gabriella@science.unideb.hu	M404
Dr. László Remete	Assistant Professor	remete.laszlo@science.unideb.hu	M406
Ms. Tímea Arnóczki	PhD student	arnoczki.timea@science.unideb.hu	M404
Ms. Orsolya Herendi	PhD student	herendi.orsolya@science.unideb.hu	M407
Mr. Ágoston Papp	PhD student	papp.agoston@science.unideb.hu	M408
Mr. Péter Sebestyén	PhD student	sebestyen.peter@science.unideb.hu	M408

4032 Debrecen, Egyetem tér 1, Geomathematics Building

**Department of Analysis** (home page: https://math.unideb.hu/en/introduction-department-analysis) **4032 Debrecen, Egyetem tér 1, Geomathematics Building** 

Name	Position	E-mail	room
Dr. Zoltán Boros	Associate Professor, Head of Department	zboros@science.unideb.hu	M326
Prof. Dr. Zsolt Páles	University Professor	pales@science.unideb.hu	M321
Prof. Dr. György Gát	University Professor	gat.gyorgy@science.unideb.hu	M324
Dr. Gergő Nagy	Associate Professor	nagyg@science.unideb.hu	M328
Dr. Eszter Novák- Gselmann	Associate Professor	gselmann@science.unideb.hu	M325
Dr. Borbála Fazekas	Assistant Professor	borbala.fazekas@science.unideb.hu	M325
Dr. Rezső László Lovas	Assistant Professor	lovas@science.unideb.hu	M330
Dr. Fruzsina Mészáros	Assistant Professor	mefru@science.unideb.hu	M325
Dr. Tibor Kiss	Assistant Professor	kiss.tibor@science.unideb.hu	M328
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Mr. Gábor Marcell Molnár	PhD student	molnar.gabor.marcell@science.unideb.hu	M322
Ms. Evelin Pénzes	PhD student	penzes.evelin@science.unideb.hu	M319
Mr. Norbert Tóth	PhD student	toth.norbert@science.unideb.hu	M323
Mr. Péter Tóth	PhD student	toth.peter@science.unideb.hu	M322

**Department of Geometry** (home page: https://math.unideb.hu/en/introduction-department-geometry)

Name	Position	E-mail	room
Prof. Dr. Zoltán	University Professor,	muzsnay@science.unideb.hu	M305
Muzsnay	Head of Department		
Prof. Dr. Csaba	University Professor,	csvincze@science.unideb.hu	M304
Vincze	Director of Institute		
Dr. Ágota Figula	Associate Professor	figula@science.unideb.hu	M303
Dr. Eszter Herendiné	Associate Professor	eszter.konya@science.unideb.hu	M307
Kónya			
Dr. Zoltán Kovács	Associate Professor	kovacsz@science.unideb.hu	M306
Dr. László Kozma	Associate Professor	kozma@unideb.hu	M306
Dr. Tran Quoc Binh	Senior Research	binh@science.unideb.hu	M305
	Fellow		
Dr. Zoltán Szilasi	Assistant Professor	szilasi.zoltan@science.unideb.hu	M329
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Dr. Márk Oláh	Assistant Professor	baro.emoke@science.unideb.hu	M329
Mr. Márton Kiss	PhD student	kiss.marton@science.unideb.hu	M308
Ms. Orsolya Lócska	PhD student	locska.orsolya@science.unideb.hu	M308
Ms. Anna Muzsnay	PhD student	muzsnay.anna@science.unideb.hu	M404
Ms. Emőke Báró	PhD student	olah.mark@science.unideb.hu	-
Ms. Gabriella Papp	PhD student	papp.gabriella@science.unideb.hu	-

4032 Debrecen, Egyetem tér 1, Geomathematics Building

# ACADEMIC CALENDAR

General structure of the academic semester (2 semesters/year):

Study poriod	1 <sup>st</sup> week	Registration*	1 week
$2^{nd} - 15^{th}$ week		Teaching period	14 weeks
Exam period	directly after the study period	Exams	7 weeks

\*Usually, registration is scheduled for the first week of September in the fall semester, and for the first week of February in the spring semester.

For further information please check the following link:

https://www.edu.unideb.hu/tartalom/downloads/University\_Calendars\_2024\_25/University\_calendar\_2024-2025-Faculty\_of\_Science\_and\_Technology.pdf

# THE MATHEMATICS BACHELOR PROGRAM

# Information about the Program

Name of BSc Program:	Mathematics BSc Program
Specialization available:	
Field, branch:	Science
Qualification:	Mathematician
Mode of attendance:	Full-time
Faculty, Institute:	Faculty of Science and Technology
	Institute of Mathematics
Program coordinator:	Prof. Dr. György Gát, University Professor
Duration:	6 semesters
ECTS Credits:	180

# **Objectives of the BSc program:**

The aim of the Mathematics BSc program is to train professional mathematicians who have deep knowledge on theoretical and applied mathematics that makes them capable of using their basic mathematical knowledge on the fields of engineering, economics, statistics and informatics. They are prepared to continue to study in an MSc program.

# Professional competences to be acquired

# A Mathematician:

#### a) Knowledge:

- He/she knows the basic methods of mathematics in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).

- He/she knows the basic correlations in pure mathematics, related to the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).

- He/she knows the basic correlations between different subdisciplines of mathematics.

- He/she is aware of the requirements of defining abstract concepts, he/she recognises general patterns and concepts inherited in the problems applied.

- He/she knows the requirements and basic methods of mathematical proofs.

- He/she is aware of the specific features of mathematical thinking.

#### b) Abilities:

- He/she is capable of formulating and communicating true and logical mathematical statements, as well as, how to exactly indicate their conditions and main consequences.

- He/she is capable of drawing conclusions of the qualitative type from quantitative data.

- He/she is capable of applying his/her factual knowledge acquired in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).

- He/she is capable of finding and exploring new correlations in the fields of analysis, algebra, geometry, discrete mathematics, operations research and probability theory (statistics).

- He/she is capable of going beyond the concrete forms of problems, and formulating them both in abstract and general forms for the sake of analysis and finding a solution.

- He/she is capable of designing experiments for the sake of data collection, as well as, of analysing the results achieved by the means of mathematics and informatics.

- He/she is capable of making a comparative analysis of different mathematical models.

- He/she is capable of effectively communicating the results of mathematical analyses in foreign languages, and by the means of informatics.

- He/she is capable of identifying routine problems of his/her own professional field, using the scientific literature available (library and electronic sources) and adapting their methods to find theoretical and practical solutions

### c) Attitude:

- He/she desires to enhance the scope of his/her mathematical knowledge by learning new concepts, as well as, for acquiring and developing new competencies.

- He/she aspires to apply his/her mathematical knowledge as widely as possible.

- Applying his/her mathematical knowledge, he/she aspires to get acquainted with the perceptible phenomena in the most thorough way possible, and to describe and explain the principles shaping them.

- Using his/her mathematical knowledge, he/she aspires to apply scientific reasoning.

- He/she is open to recognizing the specific problems in professional fields other than his/her own field and makes an effort to cooperate with experts of these fields, to the end of proposing a mathematical adaptation of field-specific problems.

- He/she is open to continuing professional training and development in the field of mathematics.

# d) Autonomy and responsibility:

- Using his/her basic knowledge acquired in mathematical subdisciplines, he/she is capable of formulating and analysing mathematical questions on his/her own.

- He/she responsibly assesses mathematical results, their applicability and the limits of their applicability.

- He/she is aware of the value of mathematical-scientific statements, their applicability and the limits of their applicability.

- He/she is capable of making decisions on his/her own, based on the results of mathematical analyses.

- He/she is aware that he/she must carry out his/her own professional work in line with the highest ethical standards and ensuring a high level of quality.

- He/she carries out his/her theoretical and practical research activities related to different fields of mathematics, with the necessary guidance, on his/her own.

# **Completion of the BSc Program**

#### The Credit System

Majors in the Hungarian Education System have generally been instituted and ruled by the Act of Parliament under the Higher Education Act. The higher education system meets the qualifications of the Bologna Process that defines the qualifications in terms of learning outcomes: statements of what students know and can do on completing their degrees. In describing the cycles, the framework uses the European Credit Transfer and Accumulation System (ECTS).

ECTS was developed as an instrument of improving academic recognition throughout the European Universities by means of effective and general mechanisms. ECTS serves as a model of academic recognition, as it provides greater transparency of study programs and student achievement. ECTS in no way regulates the content, structure and/or equivalence of study programs.

Regarding each major the Higher Education Act prescribes which professional fields define a certain training program. It contains the proportion of the subject groups: natural sciences, economics and humanities, subject-related subjects and differentiated field-specific subjects.

During the program students have to complete a total amount of 120 credit points. It means approximately 30 credits per semester. The curriculum contains the list of subjects (with credit points) and the recommended order of completing subjects which takes into account the prerequisite(s) of each subject. You can find the recommended list of subjects/semesters in chapter "Model Curriculum of Mathematics BSc Program".

		semesters					ECTS credit points	evaluation
	1.	2.	3.	4.	5.	6.		
	contact ho	urs, types o	f teaching	(l – lecture	, p – practio	ce), credit		
			poi	nts				
Linear algebra subject gro	oup							
Linear algebra 1.	28 l/3 cr.						5	exam
Dr. Gaál István	28 p /2 cr.							mid-semester grade
Linear algebra 2.		28 l/3 cr.					5	exam
Dr. Gaál István		28 p/2 cr.						mid-semester grade
Classical algebra subject g	roup							
Introduction to Algebra and Number	28 l/3 cr.						6	exam
Theory	42 p/2 cr.							mid-semester grade
Dr. Pintér Ákos								
Algebra 1.		28 l/3 cr.					5	exam
Dr. Remete László		28 p/2 cr.						mid-semester grade
Algebra 2.			28 l/3 cr.				5	exam
Dr. Remete László			28 p/2 cr.					mid-semester grade
Classical finite mathematic	cs subject g	group						
Number theory			28 l/3 cr.				5	exam
Dr. Hajdu Lajos			28 p/2 cr.					mid-semester grade
Combinatorics and graph theory	42 l/4 cr .						6	exam
Dr. Nyul Gábor	28 p/2 cr.							mid-semester grade
Classical analysis subject s	group							
Foundations of analysis	28  p/2 cr.						2	mid-semester grade
Dr. Lovas Rezső								
Introduction to analysis		42 l/5 cr.					8	exam
Dr. Tibor Kiss		42 p/3 cr.						mid-semester grade
Differential and integral calculus			42 l/5cr.				8	exam
Dr. Tibor Kiss			42 p/3 cr.					mid-semester grade
Sets, functions, real numbers			28 l/3 cr.				3	exam
Dr. Lovas Rezső								

# Model Curriculum of Mathematics BSc Program

Differential and integral calculus in			42 l/5 cr .			8	exam
several variables			42 p/3 cr.				mid-semester grade
Dr. Páles Zsolt							
Ordinary differential equations				28 l/4 cr.		6	exam
Dr. Gát György				28 p/2 cr.			mid-semester grade
Classical geometry subject	group		 		<u></u> u.		
Geometry 1.	28 l/3 cr.					5	exam
Dr. Vincze Csaba	28 p/2 cr.						mid-semester grade
Geometry 2.		28 l/3 cr.				5	exam
Dr. Vincze Csaba		28 p/2 cr.					mid-semester grade
Differential geometry				28 l/3 cr.		5	exam
Dr. Muzsnay Zoltán				28 p/2 cr.			mid-semester grade
Vector analysis					28 l/3 cr.	5	exam
Dr. Vincze Csaba					28 p/2 cr.		mid-semester grade
Probability theory subject	group						
Measure and integral theory			28 l/3 cr.			3	exam
Dr. Nagy Gergő						—	
Probability theory				42 l/4 cr.		6	exam
Dr. Fazekas István				28 p/2 cr.			mid-semester grade
Statistics					42 l/4 cr.	5	exam
Dr. Fazekas István					28 p/2 cr.		mid-semester grade
Informatics subject group							
Introduction to informatics	42 p/2 cr.					2	mid-semester grade
Dr. Tengely Szabolcs						_	
Progamming languages	28 p/2 cr.					2	mid-semester grade
Dr. Bazsó András							
Finite mathematical algor	ithms subj	ject group					
Algorithms		28 l/3 cr.				5	exam
Dr. Györkös-Varga Nóra		28 p/2 cr.					mid-semester grade
Applied number theory			42 l/3 cr.			3	exam
Dr. Hajdu Lajos							
Algorithms in algebra and number			42 p/3 cr.			3	mid-semester grade
theory							
Dr. Tengely Szabolcs							
Introduction to cryptography				28 l/3 cr.		5	exam
Dr. Bérczes Attila				28 p /2 cr.			mid-semester grade

Applied analysis subject g	group						
Numerical analysis	_		42 l/4 cr.			6	exam
Dr. Fazekas Borbála			28 p/2 cr.				mid-semester grade
Economic mathematics					28 l/3 cr.	5	exam
Dr. Mészáros Fruzsina					28 p/2 cr		mid-semester grade
Computer mathematics su	ubject grou	ір					
Analysis with computer					42 p/3 cr	3	mid-semester grade
Dr. Fazekas Borbála							
Computer statistics					28 p/2 cr	2	mid-semester grade
Dr. Sikolya-Kertész Kinga							
Computer geometry		42 p/	/3 cr.			3	mid-semester grade
Dr. Nagy Abris							
Optimizing subject group	1						
Linear programming		28 1/	3 cr.			5	exam
Dr. Mészáros Fruzsina		28 p/	/2 cr.				mid-semester grade
Basics of earth sciences a	nd mathem	atics subject gr	oup				
Basics of mathematics	14 p/0 cr.					0	signature
Dr.Györkös- Varga Nóra							
Classical mechanics			28 l/3 cr.			4	exam
Dr. Erdélyi Zoltán			14 p/1 cr.				
Theoretical mechanics					28 l/3 cr.	4	exam
Dr. Nagy Sándor					14 p/1 cr		
European Union studies	14 p/1 cr.					1	exam
Dr. Teperics Károly							
Basic environmental science	14  p/1 cr.					1	exam
Dr. Nagy Sándor Alex							
Thesis I.				5 cr.		5	mid-semester grade
Thesis II.					5 cr.	5	mid-semester grade
ontional courses					· · · ·		
optional courses						0	
optional courses				1			

#### Work and Fire Safety Course

According to the Rules and Regulations of the University of Debrecen, a student has to complete the online course for work and fire safety. Registration for the course and completion are necessary for obtaining the pre-degree certificate.

Students have to register for the subject MUNKAVEDELEM in the Neptun system.

They must read an online material until the end to get the signature on Neptun for the completion of the course. The number of credit points for the course is 1. The link of the online course is available on the webpage of the Faculty.

#### Physical Education

According to the Rules and Regulations of the University of Debrecen, a student has to complete Physical Education courses at least in two semesters during his/her Bachelor's training. The number of credit points for those courses is 1 per semester. Our University offers a wide range of facilities to complete them. Further information is available from the Sports Centre of the University, its website is: <u>http://sportsci.unideb.hu</u>.

#### Pre-degree Certification

A pre-degree certificate is issued by the Faculty after completion of the bachelor's (BSc) program. The pre-degree certificate can be issued if the student has successfully completed the study and exam requirements as set out in the curriculum, the requirements relating to Physical Education as set out in Section 10 in Rules and Regulations, internship (mandatory) – with the exception of preparing the thesis – and gained the necessary credit points (180). The pre-degree certificate verifies (without any mention of assessment or grades) that the student has fulfilled all the necessary study and exam requirements defined in the curriculum and the requirements for Physical Education. Students who obtained the pre-degree certificate can submit the thesis and take the final exam.

#### Thesis

Students have to choose a topic for their thesis two semesters before the expected date of finishing their studies, i.e., usually at the end of the 4th semester. They have to write it in two semesters, and they have to register for the courses 'Thesis 1' and 'Thesis 2' in two different semesters. They write the thesis with the help of a supervisor who should be a lecturer of the Institute of Mathematics. (In exceptional cases, the supervisor can be a member of another institute.)

Students are not required to present new scientific results, but they have to do some scientific work on their own. The thesis should be about 20–40 pages long and using the LaTeX document preparation system is recommended. The cover page has to contain the name of the institute,

the title of the thesis, the name and the degree program of the student, the name and the university rank of the supervisor. Beside the detailed discussion of the topic, the thesis should contain an introduction, a table of contents and a bibliography. The thesis has to be defended in the final exam.

# Final Exam

The final exam is an oral exam before a committee designated by the Director of the Institute of Mathematics and approved by the leaders of the Faculty of Science and Technology. The final exam consists of two parts: an account by the student on a certain exam question, and the defense of the thesis. The questions of the final exam comprise the compulsory courses of the Mathematics BSc Program. The student draws a random question from the list, and after a certain preparation period, gives an account on it. After this, the committee may ask questions also from other topics. The student gets three separate grades for their answers on the exam question, for the thesis and for the defense of the thesis.

# Final Exam Board

Board chair and its members are selected from the acknowledged internal and external experts of the professional field. Traditionally, it is the chair and in case of his/her absence or indisposition the vice-chair who will be called upon, as well. The board consists of – beside the chair – at least two members (one of them is an external expert), and questioners as required. The mandate of a Final Examination Board lasts for one year.

# Repeating a failed Final Exam

If any part of the final exam is failed it can be repeated according to the rules and regulations. A final exam can be retaken in the forthcoming final exam period. If the Board qualified the thesis unsatisfactory a student cannot take the final exam and he has to make a new thesis. A repeated final exam can be taken twice on each subject.

# Diploma

The diploma is an official document decorated with the coat of arms of Hungary which verifies the successful completion of studies in the Mathematics Bachelor Program. It contains the following data: name of HEI (higher education institution); institutional identification number; serial number of diploma; name of diploma holder; date and place of his/her birth; level of qualification; training program; specialization; mode of attendance; place, day, month and year issued. Furthermore, it has to contain the rector's (or vice-rector's) original signature and the seal of HEI. The University keeps a record of the diplomas issued.

In Mathematics Bachelor Master Program the diploma grade is calculated as the average grade of the results of the followings:

- Weighted average of the overall studies at the program (A)
- Average of grades of the thesis and its defense given by the Final Exam Board (B)
- Average of the grades received at the Final Exam for the two subjects (C)

Diploma grade = (A + B + C)/3

Classification of the award on the bases of the calculated average:

Excellent	4.81 - 5.00
Very good	4.51 - 4.80
Good	3.51 - 4.50
Satisfactory	2.51 - 3.50
Pass	2.00 - 2.50

# **Course Descriptions of Mathematics BSc Program**

Title of course: Linear algebra 1. Code: TTMBE0102	ECTS Credit points: 3					
Type of teaching, contact hours						
- lecture: 2 hours/week						
- practice: -						
- laboratory: -	- laboratory: -					
Evaluation: exam						
Workload (estimated), divided into contact hours:						
- lecture: 28 hours						
- practice: -						
- laboratory: -						
- home assignment: -						
- preparation for the exam: 62 hours						
Total: 90 hours						
<b>Year, semester</b> : 1 <sup>st</sup> year, 1 <sup>st</sup> semester						
Its prerequisite(s): -						
Further courses built on it: TTMBE0103, TTMBE0607, TTMBE0	209, TTMBG0701					
Topics of course						
Basic notions in algebra. Determinants. Operations with matrices. Ve Linear mappings. Transformation of basis and coordinates. The dim the column space of matrices are equal. Sum and direct sum of subsp of linear equations. Matrix of a linear transformation. Operations Similar matrices. Eigenvalues, eigenvectors. Characteristic polynon consisting of eigenvectors.	ctor spaces, basis, dimension. ensions of the row space and baces. Factor spaces. Systems with linear transformations. hial. The existence of a basis					
Literature						
<i>Compulsory:</i> - <i>Recommended:</i> Paul R. Halmos: Finite dimensional vector spaces, Benediction Class	sics, Oxford, 2015.					
Serge Lang, Linear Algebra, Springer Science & Business Media, 20 Howard Anton and Chris Rorres, Elementary Linear Algebra, John	013. Wiley & Sons, 2010.					
Schedule:						
1 <sup>st</sup> week						
Basic concepts of algebra. Permutations and their properties.						
2 <sup>nd</sup> week						
Determinants. Expanding determinants. Laplace expansion theorem.						
3 <sup>rd</sup> week						
Operations on matrices. Matrix algebra. Multiplication theorem matrices.	of determinants. Inverse of					

Vector space, subspace, generating system, linear dependence and independence. Basis, dimension.

5<sup>th</sup> week

Linear mappings of vector spaces. Fundamental theorems on linear mappings. Transformation of bases and coordinates.

6<sup>th</sup> week

Rank of a set of vectors, rank of a matrix. Theorem on ranks. Calculating the rank of a matrix by elimination.

 $7^{th}$  week

Sum and direct sum of subspaces. Equivalent properties. Coset of subspaces. Factor spaces of vector spaces. Dimension of the factor space.

 $8^{th}$  week

Systems of linear equations. Criteria for solubility, for the uniqueness of solutions. Homogeneous systems of linear equations. Solutions space, the dimension of the solution space.

9<sup>th</sup> week

Inhomogeneous systems of linear equations. The structure of solutions. Cramer's rule Gaussian elimination.

 $10^{th}$  week

Linear mappings of vector spaces. Kernel, image. Theorem on homomorphisms. The condition of injectivity.

11<sup>th</sup> week

Linear transformations. Injective and surjective linear transformations. The matrix of a linear transformation. Calculation the image vector. The matrix of the linear transformation in a new basis.

12<sup>th</sup> week

Operations on linear transformations. Algebra of linear transformations. Similar matrices. Automorphisms.

13<sup>th</sup> week

Invariant subspaces. Eigenvector, eigenvalues of a linear transformation. Eigenspace. Eigenvectors of distinct eigenvalues. Eigenspaces of distinct eigenvalues.

14<sup>th</sup> week

Characteristic polynomial. Algebraic and geometric multiplicity of eigenvalues. Spectrum of a linear transformation. Existence of a basis consisting of eigenvectors.

#### **Requirements:**

- for a signature

If the student fail the course TTMBG0102, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. István Gaál, university professor, DSc

Type of teaching, contact hours         - lecture: -         - practice: 2 hours/week         - laboratory: -         Evaluation: mid-semester grade         Workload (estimated), divided into contact hours:         - lecture: -         - practice: 28 hours         - laboratory: -         - home assignment: -         - preparation for the exam: 32 hours         Total: 60 hours         Year, semester: 1 <sup>st</sup> year, 1 <sup>st</sup> semester         Its prerequisite(s): -         Further course built on it: -         Topics of course         Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension.         Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Ejecnvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.         Literature       Compulsory:         - Recommended:       Paul R. Halmos: Finite dimensional vector spaces. Benediction Classics, Oxford, 2015. Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.         Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.         Schedule:       I'' week         Abstract grou	Title of course: Linear algebra 1. Code: TTMBG0102	ECTS Credit points: 2	
<ul> <li>lecture: -         practice: 2 hours/week         laboratory: -     </li> <li>Evaluation: mid-semester grade         Workload (estimated), divided into contact hours:         <ul> <li>lecture: -</li> <li>practice: 28 hours</li> <li>laboratory: -</li> <li>home assignment: -</li> <li>preparation for the exam: 32 hours</li> <li>Total: 60 hours</li> </ul> </li> <li>Year, semester: 1<sup>st</sup> year, 1<sup>st</sup> semester</li> <li>Its prerequisite(s): -         <ul> <li>Further courses built on it: -</li> </ul> </li> <li>Topics of course         <ul> <li>Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors. Elementary Linear Algebra, Springer Science &amp; Business Media, 2013.</li> <li>Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley &amp; Sons, 2010.</li> <li>Schedule:         <ul> <li>If week</li> <li>Abstract groups, permutation.</li> <li>If week</li> <li>Operations on matrices.</li> <li>If week</li> <li>Determinants. Expanding determinants.</li> <li>If week</li> <li>Operations on matrices.</li> <liif li="" week<=""></liif></ul></li></ul></li></ul>	Type of teaching, contact hours		
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<ul> <li>laboratory: -</li> <li>Evaluation: mid-semester grade</li> <li>Workload (estimated), divided into contact hours:         <ul> <li>lecture: -</li> <li>practice: 28 hours</li> <li>laboratory: -</li> <li>home assignment: -</li> <li>preparation for the exam: 32 hours</li> </ul> </li> <li>Total: 60 hours</li> <li>Year, semester: 1<sup>st</sup> year, 1<sup>st</sup> semester</li> <li>Its prerequisite(s): -</li> <li>Further courses built on it: -</li> <li>Topics of course</li> <li>Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.</li> <li>Literature</li> <li>Compulsory: -</li> <li>Recommended:</li> <li>Paul R. Halmos: Finite dimensional vector spaces, Benediction Classies, Oxford, 2015. Serge Lang, Linear Algebra, Springer Science &amp; Business Media, 2013. Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley &amp; Sons, 2010.</li> <li>Schedue:         <ul> <li>I<sup>st</sup> week</li> <li>Abstract groups, permutation.</li> <li>2<sup>std</sup> week</li> <li>Operations on matrices.</li> <li>d<sup>std</sup> week</li> <li>Netweek</li> <li>Operations on matrices.</li> <li>d<sup>std</sup> week</li> <li>Operations on matrices.</li> <li>d<sup>std</sup> week</li> <li>Transformation of bases and coordinates.</li> <li>d<sup>std</sup> week</li> </ul> </li> </ul>	- practice: 2 hours/week		
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Workload (estimated), divided into contact hours:         - lecture: -         - practice: 28 hours         - laboratory: -         - home assignment: -         - preparation for the exam: 32 hours         Total: 60 hours         Year, semester: 1st year, 1st semester         Its prerequisite(s): -         Further courses built on it: -         Topics of course         Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension. Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.         Literature         Compulsory:         -         -         Recommended:         Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015. Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.         Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.         Schedule:         Ist week         Abstract groups, permutation.         2 <sup>m''</sup> week         Operations on matrices.         Veek         Ope	Evaluation: mid-semester grade		
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Total: 60 hours         Year, semester: 1 <sup>st</sup> year, 1 <sup>st</sup> semester         Its prerequisite(s): -         Further courses built on it: -         Topics of course         Basic notions in algebra. Determinants. Operations with matrices. Vector spaces, basis, dimension.         Linear mappings. Transformation of basis and coordinates. The dimensions of the row space and the column space of matrices are equal. Sum and direct sum of subspaces. Factor spaces. Systems of linear equations. Matrix of a linear transformation. Operations with linear transformations. Similar matrices. Eigenvalues, eigenvectors. Characteristic polynomial. The existence of a basis consisting of eigenvectors.         Literature         Compulsory:         -         -         Recommended:         Paul R. Halmos: Finite dimensional vector spaces, Benediction Classics, Oxford, 2015. Serge Lang, Linear Algebra, Springer Science & Business Media, 2013.         Howard Anton and Chris Rorres, Elementary Linear Algebra, John Wiley & Sons, 2010.         Schedule:         I <sup>st</sup> week         Abstract groups, permutation.         2 <sup>std</sup> week         Determinants. Expanding determinants.         3 <sup>std</sup> week         Operations on matrices.         4 <sup>sth</sup> week         Inverse of matrices. Vectors spaces. Basis, dimension.         5 <sup>sth</sup> week         Transformation of bases and coordinates.	- preparation for the exam: 32 hours		
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$3^{rd}$ week Operations on matrices. $4^{th}$ week Inverse of matrices. Vectors spaces. Basis, dimension. $5^{th}$ week Transformation of bases and coordinates. $6^{th}$ week	Determinants. Expanding determinants.		
Operations on matrices. $4^{th}$ week Inverse of matrices. Vectors spaces. Basis, dimension. $5^{th}$ week Transformation of bases and coordinates. $6^{th}$ week	3 <sup>rd</sup> week		
$4^{th}$ week Inverse of matrices. Vectors spaces. Basis, dimension. $5^{th}$ week Transformation of bases and coordinates. $6^{th}$ week	Operations on matrices.		
Inverse of matrices. Vectors spaces. Basis, dimension. 5 <sup>th</sup> week Transformation of bases and coordinates. 6 <sup>th</sup> week	4 <sup>th</sup> week		
$5^{th}$ week Transformation of bases and coordinates. $6^{th}$ week	Inverse of matrices. Vectors spaces. Basis, dimension.		
Transformation of bases and coordinates. $6^{th}$ week	5 <sup>th</sup> week		
6 <sup>th</sup> week	Transformation of bases and coordinates.		
	6 <sup>th</sup> week		

Rank of a matrix. Calculating the rank of a matrix by elimination.

7<sup>th</sup> week

First test.

 $8^{th}$  week

Homogeneous systems of linear equations. Solutions space.

9<sup>th</sup> week

Inhomogeneous systems of linear equations. Cramer's rule Gaussian elimination.

10<sup>th</sup> week

Linear mappings of vector spaces. Calculating the kernel and image.

11<sup>th</sup> week

The matrix of a linear transformation. The matrix of the linear transformation in a new basis.

12<sup>th</sup> week

Operations on linear transformations. Similar matrices.

13<sup>th</sup> week

Able to calculate eigenvalues, eigenvectors, basis consisting of eigenvectors.

14<sup>th</sup> week

Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. -an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. István Gaál, university professor, DSc

Title of course: Linear algebra 2. Code: TTMBE0103	ECTS Credit points: 3	
Type of teaching, contact hours		
- lecture: 2 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 28 hours		
- practice: -		
- laboratory: -		
- nome assignment: -		
Total: 90 hours		
Vaar samastar: 1 <sup>st</sup> year 2 <sup>nd</sup> samastar		
I cal, semesteri. 1 year, 2 semester       Ita propognicita(a): TTMDE0102		
Its prerequisite(s): 11MBE0102		
Further courses built on it: -		
Topics of course		
Euclidean spaces. Orthonormal bases. Gram-Schmidt orthogonalization method. Orthogonal complement of a subspace. Complex vectorspaces with inner product: unitary spaces. Linear forms, bilinear forms and inner products. Adjoint of a linear transformation. Properties of the adjoint transformation. Self-adjoint transformations. Isometric/orthogonal transformations. Normal transformations.		
Literature		
Compulsory:		
- <i>Recommended:</i> Paul R. Halmos: Finite dimensional vector spaces, Benediction Class Serge Lang, Linear Algebra, Springer Science & Business Media, 20 Howard Anton and Chris Rorres, Elementary Linear Algebra, John V	sics, Oxford, 2015. )13. Wiley & Sons, 2010.	
Schedule:		
1 <sup>st</sup> week		
Nilpotent transformations. Canonical form of a nilpotent matrix.		
2 <sup>nd</sup> week		
Jordan normal form, Jordan blocks, canonical basis.		
3 <sup>ra</sup> week		
Linear forms, bilinear forms, quadratic forms.		
Canonical form of bilinear and quadratic forms. Lagrange theorem theorem. Positive definite quadratic forms and their characterization.	n. Sylvester theorem. Jacobi	
5 <sup>th</sup> week	inequality Minkowski ina	
Inner product, Euclidean space, Cauchy-Bunyakovszkij-Schwarz	inequality, willkowski me-	
23		

quality.

 $6^{th}$  week

Gram-Schmidt orthogonalization method, orthonormed bases, orthogonal complement of a subspace, Bessel inequality, Parseval equation.

 $7^{th}$  week

Bilinear and quadratic forms in complex vector spaces. Inner product. Unitary spaces.

 $8^{th}$  week

Linear, bilinear forms and inner products. Adjoint transformations. Properties of the adjoint transformation.

9<sup>th</sup> week

Self-adjoint transformations, eigenvalues, eigenvectors, canonical form.

10<sup>th</sup> week

Orthogonal transformations. Equivalent properties. Properties of orthogonal matrices.

11<sup>th</sup> week

Orthogonal transformations of Euclidean spaces. Quasi diagonal matrices. Representation of linear transformations by self-adjoint transformations.

12<sup>th</sup> week

Normal transformations in unitary spaces. Polar representation theorem.

13<sup>th</sup> week

Curves of second order, Asymptote directions. Diameters conjugated to a direction. Principal axis. Transformation to principal axis.

14<sup>th</sup> week

Application of symbolic algebra packages in linear algebra calculations.

# **Requirements:**

- for a signature

If the student fail the course TTMBG0103, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. István Gaál, university professor, DSc

Title of course: Linear algebra 2.         Code: TTMBG0103	ECTS Credit points: 2	
Type of teaching, contact hours		
- lecture: -		
- practice: 2 hours/week		
- laboratory: .		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 28 hours		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 32 hours		
Total: 60 hours		
Year, semester: 1 <sup>st</sup> year, 2 <sup>nd</sup> semester		
Its prerequisite(s): TTMBE0102		
Further courses built on it: -		
Topics of course		
Euclidean spaces. Orthonormal bases. Gram-Schmidt orthogonalization method. Orthogonal complement of a subspace. Complex vectorspaces with inner product: unitary spaces. Linear forms, bilinear forms and inner products. Adjoint of a linear transformation. Properties of the adjoint transformation. Self-adjoint transformations. Isometric/orthogonal transformations. Normal transformations.		
Literature		
Compulsory:		
<i>Recommended:</i> Paul R. Halmos: Finite dimensional vector spaces, Benediction Class Serge Lang, Linear Algebra, Springer Science & Business Media, 2 Howard Anton and Chris Rorres, Elementary Linear Algebra, John	ssics, Oxford, 2015. 013. Wiley & Sons, 2010.	
Schedule:		
1 <sup>st</sup> week		
Nilpotent transformations.		
$2^{nd}$ week		
Jordan normal form.		
3 <sup>rd</sup> week		
Linear forms, bilinear forms, quadratic forms.		
4 <sup>th</sup> week		
Canonical form of bilinear and quadratic forms. Positive definit characterization.	te quadratic forms and their	
5 <sup>th</sup> week		
Inner product, Euclidean space.		
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6<sup>th</sup> week

Gram-Schmidt orthogonalization method, orthonormed bases.

 $7^{th}$  week

First test.

 $8^{th}$  week

Adjoint transformations. Properties of the adjoint transformation.

9<sup>th</sup> week

Self-adjoint transformations, eigenvalues, eigenvectors, canonical form.

 $10^{th}$  week

Orthogonal transformations.

11<sup>th</sup> week

Orthogonal transformations of Euclidean spaces. Quasi diagonal matrices.

12<sup>th</sup> week

Normal transformations in unitary spaces. Polar representation theorem.

13<sup>th</sup> week

Curves of second order. Transformation to principal axis.

14<sup>th</sup> week

### Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

### - for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the <u>following table</u>:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. *-an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. István Gaál, university professor, DSc

<b>Title of course</b> : Introduction to algebra and number theory <b>Code</b> : TTMBE0101	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
<b>Year, semester</b> : 1 <sup>st</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): -	
Further courses built on it: TTMBE0104, TTMBG0701	

#### **Topics of course**

Relations, algebraic structures, operations and their properties. Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm. Congruence relation and congruence classes in Z, rings of congruence classes. The theorem of Euler-Fermat. Linear congruences. Linear congruence systems, Chinese remainder theorem. Two-variable and multivariate linear Diophantine equations. Peano axioms, N, Z, Q. Complex numbers, operations, conjugate, absolute value. Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity. Polynomial ring over field. Euclidean division, greatest common divisor. Polynomial rings over Z, Q, R, and C, absolute value. Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations. Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomials.

#### Literature

Compulsory:

-

*Recommended:* I. Nivan, H. S. Zuckerman, H. L. Montgomery: An introduction to the theory of numbers. John Wiley and Sons, 1991.

L. N., Childs:: A concrete introduction to higher algebra. New York, Springer, 2000.

#### Schedule:

1<sup>st</sup> week

Relations, algebraic structures, operations and their properties.

 $2^{nd}$  week

Peano axioms, natural numbers.

3<sup>rd</sup> week

Integer and rational numbers.

4<sup>th</sup> week

Complex numbers, operations, conjugate, absolute value.

5<sup>th</sup> week

Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity.

 $6^{th}$  week

Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm.  $7^{th}$  week

Congruence relation and congruence classes in Z, rings of congruence classes. Euler's phifunction, the theorem of Euler-Fermat.

 $8^{th}$  week

Linear congruences. Condition of solvability, number of solutions. Linear congruence systems, Chinese remainder theorem.

9<sup>th</sup> week

Two-variable linear Diophantine equations, condition of solvability and their connection with linear congruences, multivariate linear Diophantine equations.

 $10^{th}$  week

Polynomial ring over field. Euclidean division, greatest common divisor.

11<sup>th</sup> week

Ring of Z[x], Q[x], R[x], C[x], irreducible factorization.

12<sup>th</sup> week

Fundamental theorem of algebra. Partial fraction expression.

13<sup>th</sup> week

Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations.

14<sup>th</sup> week

Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomial.

# **Requirements:**

- for a signature

If the student fail the course TTMBG0101, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Ákos Pintér, university professor, DSc

Lecturer: Prof. Dr. Ákos Pintér, university professor, DSc

<b>Title of course</b> : Introduction to algebra and number theory <b>Code</b> : TTMBG0101	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
<b>Year, semester</b> : 1 <sup>st</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): -	
Further courses built on it: -	

#### **Topics of course**

Relations, algebraic structures, operations and their properties. Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm. Congruence relation and congruence classes in Z, rings of congruence classes. The theorem of Euler-Fermat. Linear congruences. Linear congruence systems, Chinese remainder theorem. Two-variable and multivariate linear Diophantine equations. Peano axioms, N, Z, Q. Complex numbers, operations, conjugate, absolute value. Trigonometric form of complex numbers, theorem of Moivre, nth roots of complex numbers, roots of unity. Polynomial ring over field. Euclidean division, greatest common divisor. Polynomial rings over Z, Q, R, and C, absolute value. Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations. Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomials.

#### Literature

Compulsory:

-

*Recommended:* I. Nivan, H. S. Zuckerman, H. L. Montgomery: An introduction to the theory of numbers. John Wiley and Sons, 1991.

L. N., Childs:: A concrete introduction to higher algebra. New York, Springer, 2000.

#### Schedule:

1<sup>st</sup> week

Relations, algebraic structures, operations and their properties.

 $2^{nd}$  week

Peano axioms, natural numbers.

3<sup>rd</sup> week

Integer and rational numbers.

4<sup>th</sup> week

Complex numbers, operations, conjugate, absolute value.

5<sup>th</sup> week

Trigonometric form of complex numbers, theorem of Moivre,  $n^{th}$  roots of complex numbers, roots of unity.

 $6^{th}$  week

Divisibility and division with remainder in Z. Greatest common divisor, Euclidean algorithm.  $7^{th}$  week

First test.

8<sup>th</sup> week

Euler's phi-function, the theorem of Euler-Fermat. Linear congruences. Condition of solvability, number of solutions. Linear congruence systems, Chinese remainder theorem.

9<sup>th</sup> week

Two-variable linear Diophantine equations, condition of solvability and their connection with linear congruences, multivariate linear Diophantine equations.

10<sup>th</sup> week

Polynomial ring over field. Euclidean division, greatest common divisor.

11<sup>th</sup> week

Ring of Z[x], Q[x], R[x], C[x], irreducible factorization.

12<sup>th</sup> week

Fundamental theorem of algebra. Partial fraction expression. Algebraic equations, discriminant, resultant, multiple roots, cubic and quartic equations.

13<sup>th</sup> week

Multivariate polynomials, symmetric and elementary symmetric functions, fundamental theorem of symmetric polynomial.

14<sup>th</sup> week

Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0-50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. *-an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Ákos Pintér, university professor, DSc

Lecturer: Prof. Dr. Ákos Pintér, university professor, DSc

Title of course: Algebra 1. Code: TTMBE0104	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 1 <sup>st</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0101	
Further courses built on it: TTMBE0105	

#### **Topics of course**

Definition of groups, examples. Permutations, sign of permutations. Homomorphisms. Order, cyclic groups. Subgroups, generated subgroups, Lagrange's theorem. Direct product, the fundamental theorem of finite Abelian groups. Permutation groups and group actions, Cayley's theorem. Homomorphisms and normal subgroups, conjugation. Factor groups. Homomorphism theorems. Isomorphism theorems. Basic properties of p-groups, center. Definition of rings, examples. Subrings, generated subrings. Finite rings without zero divisors. Homomorphisms and ideals, factor rings. Rings of polynomials. Euclidean rings and principal ideal domains, the fundamental theorem of number theory. Fields, simple algebraic extensions. Minimal polynomial. The multiplicativity formula for degrees. Algebraic numbers. Construction of the splitting field. Characteristics, prime fields. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials over Zp with arbitrary degree. Geometric constructions with compass and straightedge: The impossibility of doubling a cube (a. k. a. the Delian problem), trisecting an edge and squaring a circle.

#### Literature

Compulsory:

-D

Recommended:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989. Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

#### Schedule:

1<sup>st</sup> week

Groups: definition, basic properties, examples. Permutations, sign. Homomorphisms.

2<sup>nd</sup> week

Order, cyclic groups, fundamental properties.

3<sup>rd</sup> week

Subgroups, generated subgroups, Lagrange's theorem.

4<sup>th</sup> week

Direct product, fundamental theorem of finite Abelian groups (without proof). Permutation groups and group actions, Cayley's theorem.

 $5^{th}$  week

Homomorphisms and normal subgroups, conjugation. Factor group. Homomorphism theorem.  $6^{th}$  week

Isomorphism theorems (without proof). Fundamental properties of p-groups, non-triviality of the center.

 $7^{th}$  week

First test.

 $8^{th}$  week

Rings: definition, basic properties, examples. Subrings, generated subrings,. Finite rings with no zero divisors are division rings.

9<sup>th</sup> week

Homomorphisms and ideals, factor rings and their subrings. Polynomial rings.

10<sup>th</sup> week

Euclidean domains and PIDs, basic number theoretical properties. Fundamental theorem of number theory in Euclidean domains.

11<sup>th</sup> week

Fields, simple field extensions by an algebraic element. Minimal polynomial and degree of simple extensions. Field extensions by more than one elements.

12<sup>th</sup> week

The multiplicativity formula for degrees. Algebraic numbers, the field of algebraic numbers is algebraically closed. Construction of the quotient field (without the proof of uniqueness).

13<sup>th</sup> week

Characteristic of a field, prime field. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials of arbitrary degree over the p-element field. Geometric constructions: the Delean problem, trisection of an angle and squaring a circle are unsolvable with a compass and a straightedge.

14<sup>th</sup> week

Second test.

#### **Requirements:**

- for a signature

If the student fail the course TTMBG0104, then the signature is automatically denied. *- for a grade* 

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 – 39	fail (1)
40 - 49	pass (2)
50 - 59	satisfactory (3)
60 - 69	good (4)
70 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. László Remete, assistant professor, PhD

Lecturer: Dr. László Remete, assistant professor, PhD

Title of course: Algebra 1.Code: TTMBG0104	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
Year, semester: 1 <sup>st</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0101	
Further courses built on it: TTMBE0105, TTMBG0105	

#### **Topics of course**

Definition of groups, examples. Permutations, sign of permutations. Homomorphisms. Order, cyclic groups. Subgroups, generated subgroups, Lagrange's theorem. Direct product, the fundamental theorem of finite Abelian groups. Permutation groups and group actions, Cayley's theorem. Homomorphisms and normal subgroups, conjugation. Factor groups. Homomorphism theorem. Isomorphism theorems. Basic properties of p-groups, center. Definition of rings, examples. Subrings, generated subrings. Finite rings without zero divisors. Homomorphisms and ideals, factor rings. Rings of polynomials. Euclidean rings and principal ideal domains, the fundamental theorem of number theory. Fields, simple algebraic extensions. Minimal polynomial. The multiplicativity formula for degrees. Algebraic numbers. Construction of the splitting field. Characteristics, prime fields. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials over Zp with arbitrary degree. Geometric constructions with compass and straightedge: The impossibility of doubling a cube (a. k. a. the Delian problem), trisecting an edge and squaring a circle.

#### Literature

Compulsory:

*Recommended:* 

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989. Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

#### Schedule:

1<sup>st</sup> week

Groups: definition, basic properties, examples. Permutations, sign. Homomorphisms.

2<sup>nd</sup> week

Order, cyclic groups, fundamental properties.

3<sup>rd</sup> week

Subgroups, generated subgroups, Lagrange's theorem.

4<sup>th</sup> week

Direct product, fundamental theorem of finite Abelian groups (without proof). Permutation groups and group actions, Cayley's theorem.

 $5^{th}$  week

Homomorphisms and normal subgroups, conjugation. Factor group. Homomorphism theorem.  $6^{th}$  week

Isomorphism theorems (without proof). Fundamental properties of p-groups, non-triviality of the center.

 $7^{th}$  week

Students can ask questions and get an overview on the subject material in all topics prior to the first test.

 $8^{th}$  week

Rings: definition, basic properties, examples. Subrings, generated subrings,. Finite rings with no zero divisors are division rings.

9<sup>th</sup> week

Homomorphisms and ideals, factor rings and their subrings. Polynomial rings.

 $10^{th}$  week

Euclidean domains and PIDs, basic number theoretical properties. Fundamental theorem of number theory in Euclidean domains.

11<sup>th</sup> week

Fields, simple field extensions by an algebraic element. Minimal polynomial and degree of simple extensions. Field extensions by more than one elements.

12<sup>th</sup> week

The multiplicativity formula for degrees. Algebraic numbers, the field of algebraic numbers is algebraically closed. Construction of the quotient field (without the proof of uniqueness).

13<sup>th</sup> week

Characteristic of a field, prime field. Construction of finite fields, primitive roots, subfields of finite fields. Existence of irreducible polynomials of arbitrary degree over the p-element field. Geometric constructions: the Delean problem, trisection of an angle and squaring a circle are unsolvable with a compass and a straightedge.

14<sup>th</sup> week

Students can ask questions and get an overview on the subject material in all topics prior to the second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The preliminary requirement to pass the course is to obtain at least 51 percent of total points from short tests written on a once a week basis with the exception of the 1<sup>st</sup>, 7<sup>th</sup> and 14<sup>th</sup> week.

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the <u>following table</u>:

Total Score (%)	Grade		
0-39	fail (1)		
40 - 49	pass (2)		
	50 - 59	satisfactory (3)	
-------------------------------------------------------------------------------------	----------	------------------	--
	60 - 69	good (4)	
	70 - 100	excellent (5)	
If a student fail to pass at first attempt, then a retake of the tests is possible.			
-an offered grade:			
It is not possible to obtain an offered grade in this course.			

Person responsible for course: Dr. László Remete, assistant professor, PhD

Lecturer: Dr. László Remete, assistant professor, PhD

Title of course: Algebra 2. Code: TTMBE0105	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact ho	urs:
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBE0104	
Further courses built on it:	
Topics of course	

# Sylow's theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck's theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem. Algebras, minimal polynomial over algebras. Frobenius' theorem. Splitting field, existence, uniqueness, algebraic closure, existence. Normal extensions, extensions of perfect fields are simple. Galois theory. Funadamental theorem of algebra. Compass and straightedge constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

## Literature

Compulsory:

Recommended:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989. Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

## Schedule:

1<sup>st</sup> week

Sylow's theorems. Semidirect products.

 $2^{nd}$  week

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

3<sup>rd</sup> week

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

4<sup>th</sup> week

Free groups, generators, relations, Dyck's theorem. 5<sup>th</sup> week Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. 6<sup>th</sup> week Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem. 7<sup>th</sup> week First test.  $8^{th}$  week Algebras, minimal polynomial over algebras, Frobenius' theorem. 9<sup>th</sup> week Splitting field, existence, uniqueness, algebraic closure existence, uniqueness. 10<sup>th</sup> week Normal extensions, finite extensions of perfect fields are simple. 11<sup>th</sup> week Fundamental theorem of Galois theory. 12<sup>th</sup> week Fundamental theorem of algebra. Compass and straightedge constructions. 13<sup>th</sup> week Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials. 14<sup>th</sup> week Second test. **Requirements:** - for a signature If the student fail the course TTMBG0105, then the signature is automatically denied. - for a grade

The course ends in oral examination. The grade is given according to the following table:

<b>Total Score (%)</b>	Grade
0 – 39	fail (1)
40 - 49	pass (2)
50 - 59	satisfactory (3)
60 - 69	good (4)
70 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. László Remete, assistant professor, PhD

Lecturer: Dr. László Remete, assistant professor, PhD

Title of course: Algebra 2. Code: TTMBG0105	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hou	irs:
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBE0104	
Further courses built on it: -	

### **Topics of course**

Sylow's theorems. Semidirect product. Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points. Free groups, generators, relations, Dyck's theorem. Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions. Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem. Algebras, minimal polynomial over algebras. Frobenius' theorem. Splitting field, existence, uniquencess, algebraic closure, existence. Normal extensions, extensions of perfect fields are simple. Galois theory. Funadamental theorem of algebra. Compass and straightedge constructions. Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

## Literature

Compulsory:

Recommended:

John B. Fraleigh: A first course in abstract algebra, Addison-Wesley Publishing Company, 1989. Derek J. S. Robinson: A course in the theory of groups, Sringer-Verlag, 1980.

#### Schedule:

1<sup>st</sup> week

Sylow's theorems. Semidirect products.

 $2^{nd}$  week

Maximal subgroups of p-groups are normal of index p. Characteristic subgroup, commutator. Fundamental theorem of finite Abelian groups.

3<sup>rd</sup> week

Isomorphism theorems. Solvable groups and their basic properties. Alternating group is simple if acting on at least 5 points.

4<sup>th</sup> week

Free groups, generators, relations, Dyck's theorem.

5<sup>th</sup> week

Necessary and sufficient condition for a ring to be a unique factorization domain, ascending and descending chain conditions.

6<sup>th</sup> week

Field of fractions. Artinian and Noetherian rings, Hilbert's basis theorem.

7<sup>th</sup> week

Students can ask questions and get an overview on the subject material in all topics prior to the first test.

 $8^{th}$  week

Algebras, minimal polynomial over algebras, Frobenius' theorem.

9<sup>th</sup> week

Splitting field, existence, uniqueness, algebraic closure existence, uniqueness.

 $10^{th}$  week

Normal extensions, finite extensions of perfect fields are simple.

11<sup>th</sup> week

Fundamental theorem of Galois theory.

12<sup>th</sup> week

Fundamental theorem of algebra. Compass and straightedge constructions.

13<sup>th</sup> week

Theorem of Abel and Ruffini, Casus Irreducibilis is unavoidable for degree three polynomials.

14<sup>th</sup> week

Students can ask questions and get an overview on the subject material in all topics prior to the second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

# - for a grade

The preliminary requirement to pass the course is to obtain at least 51 percent of total points from short tests written on a once a week basis with the exception of the  $1^{st}$ ,  $7^{th}$  and  $14^{th}$  week.

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

<b>Total Score (%)</b>	Grade
0 – 39	fail (1)
40 - 49	pass (2)
50 - 59	satisfactory (3)
60 - 69	good (4)
70 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. -an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. László Remete, assistant professor, PhD

Lecturer: Dr. László Remete, assistant professor, PhD

<b>Title of course</b> : Number theory <b>Code</b> : TTMBE0106	ECTS Credit points: 3
Type of teaching, contact hours	i
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBG001, TTMBE0101	
Further courses built on it: TTMBE0109, TTMBG0	110
Transferration	

#### **Topics of course**

Orders of elements, generators and their description in Zp. Quadratic residues modulo p. Residues of higher degree. Arithmetic functions. Additive and multiplicative functions, some important arithmetic functions. Summatory function and Mobius-transform of arithmetic functions. The infinitude of the set of primes. Famous problems concerning primes. Primes in arithmetic progressions, the theorem of Dirichlet. The sume of the reciprocals of primes. The  $\Pi(x)$  function, the prime number theorem. Lattices, the theorems of Blichfeldt and Minkowski and their applications. The Waring problem. Pythagorean triples. Algebraic numbers, algebraic integers. The field of algebraic numbers and the ring of algebraic integers. Algebraic number fields. Degree, basis, integers and units of a number fields. Quadratic number fields and their representations in the form  $Q(\sqrt{d})$ .

# Literature

Compulsory:

Recommended:

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

K. Ireland, M. Rosen, A classical introduction to modern number theory (Second edition), Springer-Verlag.

Tom Apostol, Introduction to Analytic Number Theory, Springer-Verlag.

## Schedule:

1<sup>st</sup> week

Order of an element, generators and their description in Z<sub>p</sub>.

2<sup>nd</sup> week

Quadratic residues modulo p. Legendre- and Jacobi-symbol and their properties. Quadratic reciprocity. Congruences of higher order.  $3^{rd}$  week Number theoretical functions. Basic properties of additive and multiplicative functions.  $4^{th}$  week

Some important number theoretical functions, main properties and explicit formulas.

5<sup>th</sup> week

Summation and Mobius-transform of number theoretical functions. Mobius inversion theorem and its application to multiplicative functions.

6<sup>th</sup> week

The infinitude of the set of primes. Famous problems concerning primes: twin primes, Mersenneprimes, Fermat-primes, Goldbach's problems.

 $7^{th}$  week

Primes in arithmetic progressions, Dirichlet's theorem, special cases, the Green-Tao theorem. The divergence of the sum of the reciprocals of primes.

8<sup>th</sup> week

The behavior of the  $\Pi(x)$  function, estimates for  $\Pi(x)$ , the theorem of Chebishev. The density of the sequence of primes. The prime number theorem. Consequences, the asymptotic behavior of the n-th prime. The existence of arbitrarily long intervals containing no primes.

 $9^{th}$  week

Lattices in  $\mathbb{R}^n$ . Bases of a lattice, unimodular transformations and unimodular matrices. The fundamental parallelepiped and the lattice determinant. Lattices as discrete subgroups of  $\mathbb{R}^n$ .

 $10^{th}$  week

The theorems of Blichfeldt and Minkowski, and their applications for systems of linear Diophantine inequalities.

11<sup>th</sup> week

The Waring problem. Representing positive integers as sums of squares and higher powers. Pithagorean triples and reduced Pithagorean triples, Fermat's equation.

12<sup>th</sup> week

Algebraic numbers, algebraic integers. Degree, algebraic conjugates. The defining monic polinomial of an algebraic number and its properties. The field of the algebraic numbers, the ring of the algebraic integers. Transcendental numbers.

13<sup>th</sup> week

Algebraic number fields. Degree, basis, ring of integers, group of units.

14<sup>th</sup> week

Quadratic number fields and their representation in the form  $Q(\sqrt{d})$ . Norm and its properties in imaginary quadratic fields. Euklidean division in the rings of Gauss integers and Euler integers. Example for a ring having no unique factorization.

# **Requirements:**

- for a signature

If the student fail the course TTMBG0106, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

<b>Total Score (%)</b>	Grade
0-50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)
, day	

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Lajos Hajdu, university professor, DSc

Lecturer: Prof. Dr. Lajos Hajdu, university professor, DSc

<b>Title of course</b> : Number theory <b>Code</b> : TTMBG0106	ECTS Credit points: 2	
Type of teaching, contact hours		
- lecture: -		
- practice: 2 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 28 hours		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 32 hours		
Total: 60 hours		
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): TTMBG001, TTMBE0101		
Further courses built on it: -		
Topics of course		

Orders of elements, generators and their description in Zp. Quadratic residues modulo p. Residues of higher degree. Arithmetic functions. Additive and multiplicative functions, some important arithmetic functions. Summatory function and Mobius-transform of arithmetic functions. The infinitude of the set of primes. Famous pronblems concerning primes. Primes in arithmetic progressions, the theorem of Dirichlet. The sume of the reciprocals of primes. The  $\Pi(x)$  function, the prime number theorem. Lattices, the theorems of Blichfeldt and Minkowski and their applications. The Waring problem. Pithagorean triples. Algebraic numbers, algebraic integers. The field of algebraic numbers and the ring of algebraic integers. Algebraic number fields. Degree, basis, integers and units of a number fields. Quadratic number fields and their representations int he form  $Q(\sqrt{d})$ .

## Literature

Compulsory:

Recommended:

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

K. Ireland, M. Rosen, A classical introduction to modern number theory (Second edition), Springer-Verlag.

Tom Apostol, Introduction to Analytic Number Theory, Springer-Verlag.

## Schedule:

1<sup>st</sup> week

Order of an element, generators and their description in Z<sub>p</sub>.

2<sup>nd</sup> week

Quadratic residues modulo p. Legendre- and Jacobi-symbol and their properties. Quadratic reciprocity. Congruences of higher order.  $3^{rd}$  week Number theoretical functions. Basic properties of additive and multiplicative functions.  $4^{th}$  week

Some important number theoretical functions, main properties and explicit formulas.

5<sup>th</sup> week

Summation and Mobius-transform of number theoretical functions. Mobius inversion theorem and its application to multiplicative functions.

6<sup>th</sup> week

The infinitude of the set of primes. Famous problems concerning primes: twin primes, Mersenneprimes, Fermat-primes, Goldbach's problems. Primes in arithmetic progressions, Dirichlet's theorem, special cases, the Green-Tao theorem.

 $7^{th}$  week

First test.

 $8^{th}$  week

The divergence of the sum of the reciprocals of primes. The behavior of the  $\Pi(x)$  function, estimates for  $\Pi(x)$ , the theorem of Chebishev. The density of the sequence of primes. The prime number theorem. Consequences, the asymptotic behavior of the n-th prime. The existence of arbitrarily long intervals containing no primes.

9<sup>th</sup> week

Lattices in  $\mathbb{R}^n$ . Bases of a lattice, unimodular transformations and unimodular matrices. The fundamental parallelepiped and the lattice determinant. Lattices as discrete subgroups of  $\mathbb{R}^n$ .

 $10^{th}$  week

Theorems of Minkowski and Blichfeldt and applications concerning system of linear inequalities. *11<sup>th</sup> week* 

The Waring problem. Representing positive integers as sums of squares and higher powers. Pithagorean triples and reduced Pithagorean triples, Fermat's equation.

12<sup>th</sup> week

Algebraic numbers, algebraic integers. Degree, algebraic conjugates. The defining monic polinomial of an algebraic number and its properties. The field of the algebraic numbers, the ring of the algebraic integers. Transcendental numbers.

13<sup>th</sup> week

Algebraic number fields. Degree, basis, ring of integers, group of units. Quadratic number fields and their representation in the form  $Q(\sqrt{d})$ . Norm and its properties in imaginary quadratic fields. Euklidean division in the rings of Gauss integers and Euler integers. Example for a ring having no unique factorization.

14<sup>th</sup> week

Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

## - for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the <u>following table</u>:

Total Score (%)	Grade
0 - 60	fail (1)
61 - 70	pass (2)

	71 - 80	satisfactory (3)
	81 - 90	good (4)
	91 - 100	excellent (5)
If a student fail to pass at first attempt, then a retake of the tests is possible.		
-an offered grade:		

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Lajos Hajdu, university professor, DSc

Lecturer: Prof. Dr. Lajos Hajdu, university professor, DSc

<b>Title of course</b> : Combinatorics and graph theory <b>Code</b> : TTMBE0107	<b>ECTS Credit points:</b> 4
Type of teaching, contact hours	
- lecture: 3 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 42 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 78 hours	
Total: 120 hours	
Year, semester: 1 <sup>st</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): -	
Further courses built on it: TTMBE0606	
Topics of course	
Fundamental enumeration problems: permutations, var binomial coefficients, binomial and multinomial theo permutations cycles inclusion exclusion principle and a	riations, combinations. Properties of orem. Inversions, parity, product of

binomial coefficients, binomial and multinomial theorem. Inversions, parity, product of permutations, cycles. Inclusion–exclusion principle and applications. Basic definitions of graph theory. Eulerian trail, Hamiltonian path and cycle. Trees and forests, spanning trees, Prüfer code and Cayley's formula. Bipartite graphs. Plane graphs, dual graph, Euler's formula, planar graphs and their characterization. Vertex and edge colourings of graphs, chromatic number, the five color theorem, chromatic polynomial, chromatic index. Fundamentals of Ramsey theory. Matrices of graphs.

#### Literature

Compulsory:

Recommended:

Béla Andrásfai: Introductory Graph Theory, Akadémiai Kiadó, 1977. N. Ya. Vilenkin: Combinatorics, Academic Press, 1971. Miklós Bóna: A Walk Through Combinatorics, World Scientific, 2017.

#### Schedule:

*I<sup>st</sup> week* 

Pigeonhole principle and applications. Factorials, Stirling's formula, binomial coefficients.

 $2^{nd}$  week

Permutations, variations, combinations with and without repetitions. Properties and sums of binomial coefficients.

3<sup>rd</sup> week

Binomial and multinomial theorem. Inversions, parity, product of permutations, cycles.

4<sup>th</sup> week

Inclusion-exclusion principle and applications. Basic definitions and theorems of graph theory.

5<sup>th</sup> week

Graphs with given degree sequences. Walk, trail, path, cycle, connected graph, distance.

 $6^{th}$  week

Eulerian trail, Hamiltonian path, Hamiltonian cycle, and theorems on their existence.

7<sup>th</sup> week

Trees and forests, equivalent definitions of trees. Spanning trees, spanning forests, Prüfer code, Cayley's formula.

 $8^{th}$  week

Bipartite graphs and characterization theorem. Plane graphs, dual graph, Euler's formula.

9<sup>th</sup> week

Planar graphs, Kuratowski's theorem.

 $10^{th}$  week

Vertex colourings of graphs, chromatic number and bounds. Chromatic number of planar graphs, the five and four colour theorem.

11<sup>th</sup> week

Chromatic polynomial and properties, chromatic polynomial of trees. Edge colourings of graphs, chromatic index and bounds.

12<sup>th</sup> week

Ramsey numbers: the two-colour and the multicolour case, bounds, special values.

13<sup>th</sup> week

Adjacency and incidence matrices of graphs, characterization of fundamental graph properties using these matrices.

14<sup>th</sup> week

Fundamentals of the theory of directed graphs, directed acyclic graphs.

# **Requirements:**

- for a signature

If the student fail the course TTMBG0107, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0-50	fail (1)
51 -60	pass (2)
61 - 70	satisfactory (3)
71-80	good (4)
81-100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, associate professor, PhD

Lecturer: Dr. Gábor Nyul, associate professor, PhD

Title of course: Combinatorics and graph theory   Code: TTMBG0107	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
<b>Year, semester</b> : 1 <sup>st</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): -	
Further courses built on it: -	
Topics of course	
Fundamental enumeration problems: permutations, variations, combinations. Properties of binomial coefficients, binomial and multinomial theorem. Inversions, parity, product of permutations, cycles. Inclusion–exclusion principle and applications. Basic definitions of graph theory. Eulerian trail, Hamiltonian path and cycle. Trees and forests, spanning trees, Prüfer code and Cayley's formula. Bipartite graphs. Plane graphs, dual graph, Euler's formula, planar graphs and their characterization. Vertex and edge colourings of graphs, chromatic number, the five color theorem, chromatic polynomial, chromatic index. Fundamentals of Ramsey theory. Matrices of graphs.	
Literature	
Compulsory:	
Recommended: Béla Andrásfai: Introductory Graph Theory, Akadémiai Kiadó, 1977. N. Ya. Vilenkin: Combinatorics, Academic Press, 1971. Miklós Bóna: A Walk Through Combinatorics, World Scientific, 2017.	
Schedule:	
1 <sup>st</sup> week	
Pigeonhole principle.	
2 <sup>nd</sup> week	
Elementary combinatorial exercises.	
3 <sup>rd</sup> week	
Elementary combinatorial exercises.	

 $4^{th}$  week

Combinatorial exercises under certain restrictions.

5<sup>th</sup> week

Parity, product of permutations, cycles. 6<sup>th</sup> week Binomial and multinomial theorem. 7<sup>th</sup> week First test. 8<sup>th</sup> week Inclusion-exclusion principle. 9<sup>th</sup> week Graphs with given degree sequences, Havel-Hakimi theorem. 10<sup>th</sup> week Walk, trail, path, cycle, connectedness, distance. 11<sup>th</sup> week Eulerian trail, Hamiltonian path, Hamiltonian cycle. 12<sup>th</sup> week Trees and forests, Prüfer code. 13<sup>th</sup> week Adjacency and incidence matrices of graphs. Chromatic polynomial of graphs. 14<sup>th</sup> week Second test.

## **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0-60	fail (1)
61 - 70	pass (2)
71 - 80	satisfactory (3)
81 - 90	good (4)
91 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. *-an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Gábor Nyul, associate professor, PhD

Lecturer: Dr. Gábor Nyul, associate professor, PhD

<b>Title of course</b> : Foundations of analysis <b>Code</b> : TTMBG0212	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: 32 hours	
- preparation for the exam: -	
Total: 60 hours	
Year, semester: 1 <sup>st</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): -	
Further courses built on it: TTMBE0214, TTMBG0214	, TTMBE0213
Topics of course	
Foundations of logic and set theory. Cartesian product, rel functions. Properties of operations and the order on the set greatest lower and least upper bound; the least-upper-bour Fractions and decimals. Mathematical induction. Inequalit continuum cardinality.	ations: equivalence and order relations, t of real numbers. The notion of nd property. Exponential identities. ties. Sets of numbers of countable and
Literature	
<i>Compulsory:</i> - Walter Rudin: Principles of Mathematical Analysis, McC	Graw-Hill, New York, 1976.
Schedule:	
1 <sup>st</sup> week	
Operations with sets, properties of the operations and De l	Morgan's laws.
2 <sup>nd</sup> week	
Ordered pairs, Cartesian product. Relations; domain, range of relations, properties of composition.	e and inverse of a relation. Composition
3 <sup>rd</sup> week	

Special types of relations: the notion of a function, injective, surjective and bijective functions. Connections between functions and set operations.

### 4<sup>th</sup> week

Special types of relations: equivalence relations and partitions. Ordering relations and partial orderings. Boundedness, minimum, maximum, greatest lower bound (infimum), least upper bound (supremum).

 $5^{th}$  week

Power, exponential and logarithmic functions. Exponential identities.

6<sup>th</sup> week Further exercises and problems.

7<sup>th</sup> week First mid-term test.

8<sup>th</sup> week

Proofs by mathematical induction.

### $9^{th}$ week

Inequalities containing absolute values, second order polynomials and fractions of first order polynomials.

10<sup>th</sup> week

Notable inequalities: Bernoulli's inequality, inequalities between harmonic, geometric and arithmetic means, Schwarz and Minkowski inequalities.

 $11^{th}$  week

Fractions and decimals. Conversion between ordinary fractions and decimals.

12<sup>th</sup> week

Countable sets. Sets with the cardinality of the continuum.

13<sup>th</sup> week

Further exercises and problems.

14<sup>th</sup> week

Second mid-term test.

## **Requirements:**

Participation in practical classes is compulsory. A student must attend the practical classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Students write two mid-term tests during the semester. At the end of the semester one of the two tests can be repeated. The result of the repeated test will replace the original one. The mark will be determined by the sum of the points of the two mid-term tests according to the following tabular:

Score (percent)	Grade
0—50	fail (1)
51—60	pass (2)
60—80	satisfactory (3)
81—90	good (4)
91—100	excellent (5)

In all other questions the Education and Examination Rules and Regulations of the University of

Debrecen must be consulted.

Person responsible for course: Dr. Rezső L. Lovas, assistant professor, PhD

Lecturer: Dr. Zoltán Boros, associate professor, PhD., habil.

Dr. Eszter Novák-Gselmann, associate professor, PhD., habil.

Dr. Zsolt Páles, university professor, PhD., habil., DSc.

<b>Title of course</b> : Introduction to analysis <b>Code</b> : TTMBE0214	ECTS Credit points: 5	
Type of teaching, contact hours		
- lecture: 3 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 42 hours		
- practice: -		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 78 hours		
Total: 120 hours		
Year, semester: 1st year, 2 <sup>nd</sup> semester		
Its prerequisite(s): TTMBG0212		
Further courses built on it: TTMBE0215		
Topics of course		

#### **Topics of course**

Convergence of sequences of real numbers. Relations between convergence, boundedness and monotonicity. The Bolzano-Weierstrass theorem and Cauchy's criterion for convergence. Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number. Accumulation points, lower and upper limit of sequences. Applications. Convergence of sequences of complex numbers. The Bolzano-Weierstrass-theorem and Cauchy's criterion for sequences of complex numbers. Relations between convergence and the operations. Series of complex numbers; absolute and conditional convergence. Summation of series and operations, grouping and rearranging series. Riemann's theorem. Complex geometric series; the comparison, root and ratio tests. Abel's formula; the theorems of Dirichlet, Leibniz and Abel. Cauchy product, Mertens theorem. Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy-Hadamard theorem. Elementary functions and their addition formulas. Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces. Equivalent metrics and equivalent norms. Hausdorff's criterion for compactness. Special norms of Euclidean spaces. The Bolzano-Weierstrass-theorem and the Heine-Borel theorem. Continuity and its characterization in terms of sequences in metrc spaces. Continuity and operations, the continuity of composite functions. Relations between compactness and continuity, respectively connectedness and continuity. Continuous bijections on compact sets. Uniform continuity and its characterization.

#### Literature

Compulsory:

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

2. E. Hewitt, K. R. Stromberg: Real and Abstract Analysis. Springer-Verlag, 1965.

3. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981. *Recommended:* 

#### Schedule:

1<sup>st</sup> week

Convergence of sequences of real numbers. Relations between convergence, boundedness and monotonicity. The Bolzano–Weierstrass theorem and Cauchy's criterion for convergence.  $2^{nd}$  week

Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number.

3<sup>rd</sup> week

Accumulation points, lower and upper limit of sequences. Applications.

4<sup>th</sup> week

Convergence of sequences of complex numbers. The Bolzano–Weierstrass-theorem and Cauchy's criterion for sequences of complex numbers. Relations between convergence and the operations.  $5^{th}$  week

Complex geometric series; the comparison, root and ratio tests.

6<sup>th</sup> week

Abel's formula; the theorems of Dirichlet, Leibniz and Abel. Cauchy product, Mertens theorem.  $7^{th}$  week

Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem.

8<sup>th</sup> week

Elementary functions and their addition formulas.

9<sup>th</sup> week

Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces.  $10^{th}$  week

Boundedness and uniform boundedness in metric spaces. Topology in metric spaces. Equivalent metrics and equivalent norms.

11<sup>th</sup> week

Compactness in metric spaces. Hausdorff's criterion for compactness.

12<sup>th</sup> week

Special norms of Euclidean spaces. The Bolzano-Weierstrass-theorem and the Heine-Borel theorem.

13<sup>th</sup> week

Continuity and its characterization in terms of sequences in metrc spaces. Continuity and operations, the continuity of composite functions.

14<sup>th</sup> week

Relations between compactness and continuity, respectively connectedness and continuity. Continuous bijections on compact sets. Uniform continuity and its characterization.

## **Requirements:**

The course ends in an oral or written **examination**. Two assay questions are chosen randomly from the list of assays. In case one of them is incomplete, the examination ends with a fail. In lack of the knowledge of proofs, at most satisfactory can be achieved. The grade for the examination is given according to the following table:

Score	Grade	
0-59%	fail (1)	
60-69%	pass (2)	

70-79%	satisfactory (3)
80-89%	good (4)
90-100%	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

Person responsible for course: Dr. Tibor Kiss, assistant professor, PhD

Lecturer: Dr. Tibor Kiss, assistant professor, PhD

<b>Title of course</b> : Introduction to analysis <b>Code</b> : TTMBG0214	ECTS Credit points: 3	
Type of teaching, contact hours		
- lecture: -		
- practice: 3 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 42 hours		
- laboratory: -		
- home assignment: 32 hours		
- preparation for the exam: -		
Total: 74 hours		
Year, semester: 1st year, 2 <sup>nd</sup> semester		
Its prerequisite(s): TTMBG0212		
Further courses built on it: TTMBE0214		
Topics of course		
Convergence of sequences of real numbers. Convergence and operation limit and the order. Elementary sequences; the Euler number. Co	ons, the relation between the nvergence of sequences of	

limit and the order. Elementary sequences; the Euler number. Convergence of sequences of complex numbers. Series of complex numbers; absolute and conditional convergence. Summation of series and operations, grouping and rearranging series. Complex geometric series; the comparison, root and ratio tests. Pointwise and uniform convergence of function sequences and series. Cauchy's criterion and the sufficient condition of Weiersrass for uniform convergence. Power series; the Cauchy–Hadamard theorem. Elementary functions and their addition formulas. Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Basic notions in metric spaces. Special norms of Euclidean spaces. Continuity and its characterization in terms of sequences in metric spaces.

## Literature

Compulsory:

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

2. E. Hewitt, K. R. Stromberg: Real and Abstract Analysis. Springer-Verlag, 1965.

3. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981. *Recommended:* 

#### Schedule:

1<sup>st</sup> week

Convergence of sequences of real numbers. Cauchy's criterion for convergence.

2<sup>nd</sup> week

Convergence and operations, the relation between the limit and the order. Elementary sequences; the Euler number (ratio of polynomials and exponential polynomials, difference of roots).  $3^{rd}$  week

Convergence and operations, the relation between the limit and the order. Elementary sequences;

the Euler number (n-square and n-power of ratio of linear expressions).
4 <sup>th</sup> week
Convergence of series via definition, via determining the closed form of partial sums.
5 <sup>th</sup> week
Complex geometric series; the comparison, root and ratio tests.
6 <sup>th</sup> week
Summary.
7 <sup>th</sup> week
Mid-term test.
$\delta^{th}$ week
Power series and elementary functions.
9 <sup>th</sup> week
Pointwise and uniform convergence of sequence and series of functions.
10 <sup>th</sup> week
Metric spaces, normed spaces, Banach spaces, Euclidean spaces. Special norms of Euclidean
spaces.
11 <sup>th</sup> week
Topology and compactness in metric spaces.
12 <sup>th</sup> week
Continuity and its characterization in terms of sequences in Euclidean spaces.
13 <sup>th</sup> week
Summary.
14 <sup>th</sup> week
End-term test.

## **Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor.

The course finishes with a grade, which is based on the total sum of points of the mid-term test (in the 7<sup>th</sup> week) and the end-term test (in the 14<sup>th</sup> week). One of the tests can be repeated. The final grade is given according to the following table:

	0	e
Score		Grade
)-59%		fail (1)
50-69%		pass (2)
70-79%		satisfactory (3)
80-89%		good (4)
90-100%	)	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

#### Person responsible for course: Dr. Tibor Kiss, assistant professor, PhD

Lecturer: Dr. Tibor Kiss, assistant professor, PhD

<b>Title of course</b> : Differential and integral calculus <b>Code</b> : TTMBE0215	ECTS Credit points: 5	
Type of teaching, contact hours		
- lecture: 3 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 42 hours		
- practice: -		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 78 hours		
Total: 120 hours		
Year, semester: 2nd year, 1st semester		
Its prerequisite(s): TTMBE0214, TTMBG0215		
Further courses built on it: TTMBE0216		
Topics of course		

Limit of functions and its computation using limit of sequences. Cauchy's criterions; the relation between the limit and the operations, respectively the order. The relation between limit and uniform convergence, respectively continuity and uniform convergence; Dini's theorem. Right- and left-sided limits; points of discontinuity; classification of discontinuities of the first kind; limit properties of monotone functions. Elementary limits; the introduction of pi. Functions stemming from elementary functions. Differentiability and approximation with linear functions. Differentiability and operations; the chain rule and the differentiability of the inverse function. Local extremum, Fermat principle. The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules. Higher order differentiability; Taylor's theorem, monotonicity and differentiability, higher order conditions for extrema. Convex functions. The definition of antiderivatives; basic integrals, rules of integration. The main classes of integrable functions. Inequalities, mean value theorems for the Riemann integral. The Newton–Leibniz theorem and the properties of antiderivatives. The relation between Riemann-integrability and uniform convergence. Lebesgue's criterion. Improper Riemann integral and its criteria.

#### Literature

Compulsory:

1. W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

2. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981. *Recommended:* 

#### Schedule:

1<sup>st</sup> week

Limit of functions and its computation using limit of sequences. Cauchy's criterions; the relation between the limit and the operations, respectively the order.

2<sup>nd</sup> week

The relation between limit and uniform convergence, respectively continuity and uniform convergence; Dini's theorem.

3<sup>rd</sup> week

Right- and left-sided limits; points of discontinuity; classification of discontinuities of the first kind; limit properties of monotone functions.

4<sup>th</sup> week

Elementary limits; the introduction of pi. Functions stemming from elementary functions.

5<sup>th</sup> week

Differentiability and approximation with linear functions. Differentiability and continuity; differentiability and operations; the chain rule and the differentiability of the inverse function.

6<sup>th</sup> week

Local extremum, Fermat principle. The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules. Higher order differentiability; Taylor's theorem.

7<sup>th</sup> week

Monotonicity and differentiability, higher order conditions for extrema. Convex functions.  $8^{th}$  week

The definition of antiderivatives; basic integrals, rules of integration.

9<sup>th</sup> week

Darboux integrals and their properties.

10<sup>th</sup> week

Riemann integral and its properties.

11<sup>th</sup> week

The main classes of integrable functions. Inequalities, mean value theorems for the Riemann integral. The Newton–Leibniz theorem and the properties of antiderivatives.

12<sup>th</sup> week

The relation between Riemann-integrability and uniform convergence. Applications. Improper Riemann-integral.

13<sup>th</sup> week

Lebesgue null sets. Modulus of continuity.

14<sup>th</sup> week

Lebesgue's criterion and its applications.

#### **Requirements:**

The course ends in an oral or written **examination**. Two assay questions are chosen randomly from the list of assays. In case one of them is incomplete, the examination ends with a fail. In lack of the knowledge of proofs, at most satisfactory can be achieved. The grade for the examination is given according to the following table:

Score	Grade
0-59%	fail (1)
60-69%	pass (2)
70-79%	satisfactory (3)
80-89%	good (4)
90-100%	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

**Person responsible for course:** Dr. Tibor Kiss, assistant professor, PhD

Lecturer: Dr. Tibor Kiss, assistant professor, PhD

<b>Title of course</b> : Differential and integral calculus <b>Code</b> : TTMBG0215	ECTS Credit points: 3	
Type of teaching, contact hours - lecture: - practice: 3 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 42 hours		
- laboratory: -		
- home assignment: 48 hours		
- preparation for the exam: -		
Total: 90 hours		
Year, semester: 2nd year, 1st semester		
Its prerequisite(s): TTMBE0214		
Further courses built on it: TTMBE0215		
Topics of course		
Limit of functions and its computation using limit of sequences. Differentiability and operations; the chain rule and the differentiability of the inverse function. Local extremum, Fermat principle, mean value theorems. L'Hospital rules. Higher order differentiability; Taylor's theorem. Monotonicity, convexity, extrema. Basic integrals, rules of integration. Riemann integral and the Newton–Leibniz theorem. Inequalities for Riemann integral. Improper Riemann integral.		
Literature		
<i>Compulsory:</i> 1. W. Rudin: Principles of Mathematical Analysis. McGraw- 2. K. R. Stromberg: An introduction to classical real analysis	Hill, 1964. Wadsworth, California, 1981.	

Recommended:

## Schedule:

1<sup>st</sup> week

Computing limits and derivatives of functions and its computation using limit of sequences.

2<sup>nd</sup> week

Differentiability and operations; the chain rule and the differentiability of the inverse function.

 $3^{rd}$  week Higher order differentiability; Taylor's theorem.

4<sup>th</sup> week

The mean value theorems of Rolle, Lagrange, Cauchy and Darboux. L'Hospital rules.

5<sup>th</sup> week

Monotonicity, convexity, extrema of functions.

6<sup>th</sup> week

Summary

7<sup>th</sup> week

Midterm test.

$\delta^{th}$ week
Basic integrals, rules of integration.
9 <sup>th</sup> week
Integration of partial fractions.
10 <sup>th</sup> week
Applications of the integration of partial fractions.
11 <sup>th</sup> week
Riemann sums and Riemann integral. The Newton-Leibniz theorem. Improper Riemann integrals.
12 <sup>th</sup> week
Inequalities for Riemann integral.
13 <sup>th</sup> week
Summary.
14 <sup>th</sup> week
Endterm test.

### **Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor.

The course finishes with a grade, which is based on the total sum of points of the mid-term test (in the 7<sup>th</sup> week) and the end-term test (in the 14<sup>th</sup> week). One of the tests can be repeated. The final grade is given according to the following table:

0	Ũ
Score	Grade
0-59%	fail (1)
60-69%	pass (2)
70-79%	satisfactory (3)
80-89%	good (4)
90-100%	excellent (5)

In general, the EDUCATION AND EXAMINATION RULES AND REGULATIONS have to be taken into account.

Person responsible for course: Dr. Tibor Kiss, assistant professor, PhD

Lecturer: Dr. Tibor Kiss, assistant professor, PhD

Title of course: Sets, functions, real numbers     Code: TTMBE0213	ECTS Credit points: 3	
Type of teaching, contact hours		
- lecture: 2 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 28 hours		
- practice: -		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 62 hours		
Total: 90 hours		
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): TTMBG0212		
Further courses built on it: -		
Topics of course		
Foundations of set theory. Relations. Equivalence and ord partially ordered sets and Tarski's fixed point theorem. Ca the Schröder–Bernstein theorem. Axioms of the real numb subsets of the reals: natural numbers, integers, rational an	ler relations, functions. Basic notions in ardinality of sets, Cantor's theorem and bers and their corollaries. Notable d irrational numbers. Uniqueness of the	

set of real numbers. Existence and uniqueness of the nth root. The p-adic represantion of real numbers. Notable inequalities. The field of complex numbers. Cardinality of sets of numbers.

#### Literature

Compulsory:

- Walter Rudin: Principles of Mathematical Analysis, McGraw-Hill, New York, 1976.

Schedule:

1st week

Basic notions of set theory. Axiom of empty set, axiom of extensionality, axiom of pair, axiom of union, axiom of power set. Axiom of separation, Russel's theorem. Operations with sets, properties of the operations and De Morgan's laws.

2nd week

Ordered pairs, Cartesian product. Relations; domain, range and inverse of a relation. Composition of relations, properties of composition.

3rd week

The notion of a function, injective, surjective and bijective functions. Connections between functions and set operations. Indexed families of sets, axiom of choice.

4th week

Equivalence relations and partitions. Ordering relations and partial orderings, chains and intervals. Boundedness, minimum, maximum, infimum, supremum.

#### 5th week

Completeness. Equivalent formulations of the axiom of choice: Zermelo's well ordering theorem, Hausdorff's maximum principal, Kuratowski—Zorn lemma.

## 6th week

Cardinality of sets. Comparison of cardinalities. Tarski's fixed point theorem and the Schröder-Bernstein theorem. Properties of relations of cardinalities.

### 7th week

Cardinality of a power set. Finite and infinite sets. Further axioms: axiom of regularity and axiom of infinity.

### 8th week

The axioms of real numbers. Corollaries of the field axioms and order axioms. The absolute value function. Dedekind's theorem and Cantor's theorem.

## 9th week

Natural numbers, Peano's axioms. The Archimedean property. Principle of induction and recursive definition. Properties of the binary operations. The binomial theorem and Bernoulli's inequality.

## 10th week

Integers, integer part and fractional part. Rational and irrational numbers, denseness theorems. Uniqueness of the set of real numbers.

#### 11th week

Definition and existence of nth roots. Powers with rational exponents. p-adic fractions.

#### 12th week

Notable inequalities. Power means. Inequality between the arithmetic, geometric and harmonic means. Schwarz and Minkowski inequalities.

## 13th week

The set of complex numbers and its algebraic structure. Real part, imaginary part, conjugate and absolute value of a complex number. Schwarz inequality for complex numbers.

## 14th week

Finite and infinite sets. Countable sets and the cardinality of the continuum. Cardinality of the set of natural numbers, integers, rational, real and complex numbers.

#### **Requirements:**

The course ends in an oral exam. In the exam students give an account on two exam questions. Students who reveal a profound lack of knowledge will fail the exam. Students who cannot prove the theorems in their exam questions can get at most a satisfactory (3) grade. Concerning all other questions, the Education and Examination Rules and Regulations of the University of Debrecen

must be consulted.

Person responsible for course: Dr. Rezső L. Lovas, assistant professor, PhD

Lecturer: Dr. Zoltán Boros, associate professor, PhD., habil. Dr. Eszter Novák-Gselmann, associate professor, PhD., habil.

<b>Title of course</b> : Differential and integral calculus in several variables <b>Code</b> : TTMBE0216	ECTS Credit points: 5
Type of teaching, contact hours	
- lecture: 3 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 42 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 78 hours	
Total: 120 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0215	
Further courses built on it:	

#### **Topics of course**

The Banach contraction principle. Linear maps in normed spaces. The Fréchet derivative; chain rule, differentiability and operations. The mean value inequality of Lagrange. Inverse and implicit function theorems. Further notions of derivatives; the representation of the Fréchet derivative. Continuous differentiability and continuous partial differentiability; sufficient condition for differentiability. Higher order derivatives; Schwarz–Young theorem, Taylor's theorem. Local extremum and Fermat principle; the second order conditions for extrema. The method of Lagrange Multipliers. The definition of the Riemann integral; the integral and operations, criteria for integrability, inequalities and mean value theorems for the Riemann integral. The relation between the Riemann integral and uniform convergence. Lebesgue's theorem. Fubini's theorem. Jordan measure and its properties; integration over Jordan measurable sets. Fubini's theorem on simple regions, change of variables. Functions of bounded variation, total variation, decomposition theorem of Jordan. The Riemann–Stieltjes integral and its properties. Integral. Line integral, potential function and antiderivative. Necessary and sufficient conditions for the existence of antiderivatives.

#### Literature

Compulsory:-Recommended: W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964. K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

## Schedule:

*I<sup>st</sup> week* Metric spaces. Limit of sequences and completeness. The Banach fixed point theorem. Characterization of Banach spaces among normed spaces. Compactness in normed spaces. The equivalence of the norms in finite dimensional normed spaces. Examples.

 $2^{nd}$  week The norm of linear mappings, characterizations of bounded linear maps. The structure of the space of linear maps. Convergence of Neumann series. The topological structure of invertible linear self-maps in a Banach space. The open mapping theorem and its consequences.

 $3^{rd}$  week The notion of Fréchet derivative and its uniqueness. The connection of differentiability and continuity. The Fréchet derivative of affine and bilinear maps. The chain rule and its consequences.

*4<sup>th</sup> week* The Hahn-Banach theorem for normed spaces and the Lagrange mean value inequality. Strict and continuous Fréchet differentiability. The inverse and implicit function theorems.

5<sup>th</sup> week The notions of directional and partial derivatives and their connection to Fréchet differentiability. The representation of the Fréchet derivative via partial derivatives. Sufficient condition for Fréchet differentiability, the characterization of continuous differentiability.

 $6^{th}$  week Higher-order derivatives, the Schwarz-Young theorem and the Taylor theorem. Local minimum and maximum, the Fermat principle. Characterizations of positive definite and positive semidefinite quadratic forms. The second-order necessary and sufficient conditions of optimality. Constrained optimization and the method of Lagrange Multipliers.

7<sup>th</sup> week Compact intervals in Euclidean spaces. Partitions of intervals. The lower and upper Riemann sums of bounded functions and their basic properties. The lower and upper Darboux integrals and their properties. The Darboux theorem. The additive interval property of the Darboux integrals.

 $\delta^{th}$  week The notion of Riemann integral and examples for non-integrability. The linearity and additive interval property of Riemann integral. The Riemann criterion of integrability. Further criteria of integrability.

 $9^{th}$  week Integrability and continuity. Sufficient conditions of integrability. Operations with Riemann integrable functions. Mean value theorem for the Riemann integral. Uniform convergence and integrability. The structure of the space of Riemann integrable functions.

 $10^{th}$  week Computation of the Riemann integral, the Fubini theorem and its consequences. Null sets in the sense of Lebesgue and their properties. The characterization of Riemann integrability via the Lebesgue criterion.

11<sup>th</sup> week The Jordan measure and its properties. Characterization of Jordan measurability and Jordan null sets. The Riemann integral over Jordan measurable sets. Algebraic properties, connection of integrability and continuity. The Fubini theorem on simple regions. Change of variables.

12<sup>th</sup> week Functions of bounded variation and their structure. The additive interval property of total variation, the Jordan decomposition theorem and its corollaries. The computation of the total variation.

13<sup>th</sup> week The Riemann-Stieltjes integral, its bilinearity and additive interval property. Integration by parts. Sufficient conditions for Riemann-Stieltjes integrability and the computation of the integral.

*14<sup>th</sup> week* Curves and the length of curves. The line integral of vector fields. Antiderivative (potential) of vector fields. The Newton-Leibniz theorem. Differentiation of parametric integrals. The necessary and sufficient conditions for the existence of antiderivatives.

#### **Requirements:**

- for a signature

Attendance at lectures is recommended, but not compulsory.

- for a grade

The course ends in an **examination**. Before the examination students must have grade at least 'pass' on *Differential and integral calculus in several variables* practice (TTMBG0216). The grade for the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-61	pass (2)
62-74	satisfactory (3)

75-87 88-100 good (4) excellent (5)

If the average of the score of the examination is below 50, students can take a retake examination in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Prof. Dr. Zsolt Páles, university professor, DSc

Lecturer: Prof. Dr. Zsolt Páles, university professor, DSc

<b>Title of course</b> : Differential and integral calculus in several variables <b>Code</b> : TTMBG0216	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: -	
- practice: 3 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 42 hours	
- laboratory: -	
- home assignment: 24 hours	
- preparation for the tests: 24 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0215	
Further courses built on it:	

#### **Topics of course**

The Fréchet derivative, directional derivative, partial derivative. Examples for differentiability and non-differentiability. Computation of the derivatives, chain rule. The inverse and implicit function theorems. Further notions of differentiability, the representation of the Fréchet derivative. Higher order derivatives; Schwarz–Young theorem, Taylor's theorem. Local extremum and Fermat principle; the second-order conditions for extrema. The method of Lagrange Multipliers. The computation of the Riemann integral; the integral and operations, criteria for integrability. Fubini's theorem on simple regions, change of variables. Functions of bounded variation, total variation. The Riemann–Stieltjes integral, integration by parts. The computation of the integral. Line integral; potential function and antiderivative.

## Literature

Compulsory:-

Recommended:

W. Rudin: Principles of Mathematical Analysis. McGraw-Hill, 1964.

K. R. Stromberg: An introduction to classical real analysis. Wadsworth, California, 1981.

#### Schedule:

*l<sup>st</sup> week* Limit of vector-valued functions in several variables. Checking Fréchet differentiability, directional differentiability, partial differentiability by definition.

 $2^{nd}$  week The representation of the derivative in terms of partial derivatives. Computation of the directional and partial derivatives. Applications of the chain rule.

 $3^{rd}$  week The inverse and implicit function theorems, implicit differentiation. Higher-order derivatives and differentials. Applications of the Taylor theorem.

4<sup>th</sup> week The Fermat principle for local minimum and maximum. Characterization of positive definite and positive semidefinite quadratic forms. The second-order necessary and sufficient conditions of optimality.

5<sup>th</sup> week Optimization problems with equality and inequality constraints and applications of the
method of Lagrange Multipliers.

 $6^{th}$  week Survey of the results and methods of the  $1^{st}$ - $5^{th}$  weeks.

7<sup>th</sup> week Mid-term test.

 $\delta^{th}$  week Computation of the Riemann integral using the Fubini theorem. The Jordan measure of bounded sets.

9<sup>th</sup> week Computation of the Riemann integral using change of variables.

10<sup>th</sup> week Functions of bounded and of unbounded variation. The computation of total variation.

11<sup>th</sup> week The Riemann-Stieltjes integral and the line integral.

12<sup>th</sup> week Existence and non-existence of the primitive function (potential function) of vector fields.

13<sup>th</sup> week Survey of the results and methods of the 8<sup>th</sup>-12<sup>th</sup> weeks.

14<sup>th</sup> week End-term test.

## **Requirements:**

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the  $7^{th}$  week and the end-term test in the  $14^{th}$  week. Students have to sit for the tests.

- for a grade

The minimum requirement for the average of the mid-term and end-term tests is 50%.

Score	Grade
0-49	fail (1)
50-61	pass $(2)$
62-74	satisfactory (3)
75-87	good (4)
88-100	excellent (5)

If the average of the scores of the tests is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Prof. Dr. Zsolt Páles, university professor, DSc

Lecturer: Prof. Dr. Zsolt Páles, university professor, DSc

<b>Title of course</b> : Ordinary differential equations <b>Code</b> : TTMBE0217	ECTS Credit points: 4
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 3 <sup>rd</sup> year, 1st semester	
Its prerequisite(s): TTMBE0216	
Further courses built on it:	

Differential equations solvable in an elementary way. Cauchy problem; solution, maximal solution, locally and globally unique solution. Lipschitz condition; the theorem on global-local existence and uniqueness. Continuous dependence on the initial value. The Arzelà–Ascoli theorem and Peano's theorem. First order linear systems of differential equations; fundamental matrix, Liouville's formula, variation of constants. The construction of fundamental matrices of linear systems of differential equations with constant coefficients. Higher order (linear) differential equations and the Transition Principle; Wronski determinant and Liouville's formula. Fundamental sets of solutions of higher order linear differential equations with constant coefficients. Stability; Gronwall–Bellmann lemma and the stability theorem of Lyapunov. Elements of calculus of variations: the Du Bois-Reymond lemma and the Euler–Lagrange equations. Applications.

#### Literature

*Compulsory/Recommended:* E. A. Coddington, N. Levinson: Theory of Ordinary Differential Equations. McGraw-Hill, 1955.

#### Schedule:

I<sup>st</sup> week

Ordinary explicit differential equations of first order solvable in an elementary way.

Separable, linear and exact equations. The Euler multiplicator.

## 2<sup>nd</sup> week

The notion of the Cauchy problem with respect to ordinary explicit differential equation systems of first order. Solution, complete solution, unique solution. Sufficient condition for the existence of the complete solution, global and local solvability.

#### 3<sup>rd</sup> week

Complete metric spaces. The parametric version of the Banach fixed-point theorem. Weighted

function spaces; The Cauchy problem and its equivalent integral equation.

## 4<sup>th</sup> week

Lipschitz properties. Global existence and uniqueness theorem. Continuous dependence on initial value; local existence and uniqueness theorem.

## 5<sup>th</sup> week

Compact operators; Schauder's fixed point theorems. Compact subsets of the space of continuous functions on intervals. Equicontinuity and uniform boundedness. Arzelà–Ascoli theorem.

## $6^{th}$ week

Peano's existence theorem.

#### 7<sup>th</sup> week

Linear differential equation systems of first order and their existence and uniqueness. Fundamental system and fundamental matrix; Liouville's formula. The method of constant variation.

## $8^{th}$ week

The general theory of linear differential equation systems with constant coefficients: spectral radius, expression of analytic functions of matrices, the fundamental system of linear differential equation systems of first order with constant coefficient.

#### 9<sup>th</sup> week

The general theory of explicit differential equations of higher order: transmission principle, Global existence and uniqueness theorem. Cauchy problem for higher order linear differential equations. The concept and the existence of the fundamental system; Wronski-determinant and Liouville formula.

## 10<sup>th</sup> week

Equivalent characterization of the fundamental system of a higher order linear linear differential equation. The constant variation method. The fundamental system of higher order homogeneous linear differential equations with constant coefficients.

## 11<sup>th</sup> week

Elements of stability theory. Definition of unstable, stable and asymptotically stable solution. Stability of the null-solution of homogeneous linear differential equation systems with constant coefficients.

## 12<sup>th</sup> week

The Gronwall–Bellmann lemma and the stability theorem of Lyapunov.

# 13<sup>th</sup> week

Elements of calculus of variation. The set of admissible functions and its topology. The differentiation of the perturbed basic functional and the Du-Bois-Reymond lemma.

## 14<sup>th</sup> week

The Euler-Lagrange differential equations. Applications: the problem of minimal surface solid of revolution, the Poincaré half-circle model of Bolyai–Lobachevsky's geometry. The Lagrange

discussion of classical mechanics.

#### **Requirements:**

#### - for a signature

Attendance at **lectures** is recommended, but not compulsory.

#### - for a grade

The course ends in an **examination**. Before the examination students must have grade at least 'pass' on ordinary differential equations practice (TTMBG0217).

The grade for the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-61	pass (2)
62-74	satisfactory (3)
75-87	good (4)
88-100	excellent (5)

If the average of the score of the examination is below 50, students can take a retake examination in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Prof. Dr. György Gát, university professor, DSc

Lecturer: Prof. Dr. György Gát, university professor, DSc

Title of course: Ordinary differential equationsECTS CreditCode: TTMBG0217ECTS Credit	
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: 14 hours	
- preparation for the tests: 18 hours	
Total: 60 hours	
Year, semester: 3 <sup>nd</sup> year, 1st semester	
Its prerequisite(s): Differential and integral calculus in sev	veral variables: TTMBE0216
Further courses built on it:	
Topics of course	
Differential equations solvable in an elementary way. Linear differential equation systems of first order; fundament variation. Construction of the fundamental matrix of line constant coefficients. Higher order (linear) differential ex-	ntal matrix, Liouville formula, constant ear differential equation systems with quations and transmission principles;

constant coefficients. Higher order (linear) differential equations and transmission principles; Wronski determinant and Liouville formula. Fundamental system of linear differential equations with constant coefficients. Elements of calculus variation: Du Bois-Reymond lemma and Euler-Lagrange equation.

## Literature

Compulsory/Recommended:

E. A. Coddington, N. Levinson: Theory of Ordinary Differential Equations. McGraw-Hill, 1955.

#### Schedule:

# 1<sup>st</sup> week

Differential equations solvable in an elementary way. Separable equations.

## 2<sup>nd</sup> week

Differential equations of type that can be traced back into a separable equation (linear substitution, homogeneous equations).

## $3^{rd}$ week

Types that can be traced back into a separable equation (linear fractional substitution).

## $4^{th}$ week

Differential equations that can be solved in an elementary way: first order linear equations. Bernoulli and Riccati equations.

## $5^{th}$ week

Differential equations that can be solved in an elementary way: exact equations, Euler's multipliers.

## $6^{th}$ week

Summarize, practice and deepen the foregoing.

7<sup>th</sup> week

Test

## $8^{th}$ week

First order homogeneous linear differential equation systems with constant coefficients. Construction of the fundamental system. Expression of analytic functions of matrices.

## 9<sup>th</sup> week

First order inhomogeneous linear differential equation systems with constant coefficient. The constant variation method

## 10<sup>th</sup> week

Higher order linear equations with constant coefficients. Transmission principle, Characteristic polynomial, reduced constant variation, test function.

## 11<sup>th</sup> week

Higher linear equations with variable coefficients. Wronski determinant, Liouville formula and D'Alembert reduction.

## 12<sup>th</sup> week

Elements of calculus of variation. The Euler-Lagrange differential equations.

13<sup>th</sup> week

Summarize, practice and deepen the foregoing.

14<sup>th</sup> week

#### Test

## **Requirements:**

## - for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Missed practice classes should be made up for at a later date, to be discussed with the tutor. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: the mid-term test in the 7<sup>th</sup> week and the end-term test in

the 14<sup>th</sup> week. Students have to sit for the tests.

#### - for a grade

The minimum requirement for the average of the mid-term and end-term tests is 50%. The score is the average of the scores of the two tests and the grade is given according to the following table:

Score	Grade
0-49	fail (1)
50-61	pass (2)
62-74	satisfactory (3)
75-87	good (4)
88-100	excellent (5)

If the average of the scores is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Prof. Dr. György Gát, university professor, DSc

Lecturer: Prof. Dr. György Gát, university professor, DSc

<b>Title of course</b> : Geometry 1. <b>Code</b> : TTMBE0301	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hou	irs:
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 1st year, 1st semester	
Its prerequisite(s): TTMBG0301 (p)	
Further courses built on it: Geometry 2.	

Absolute Geometry: incidence axioms, ruler postulate, plane separation postulate, protractor postulate and the axiom of congruence. Some representative results in Absolute Geometry: congruence theorems, perpendicular and parallel lines, sufficient conditions for parallelism, inequalities. The Euclidean parallel postulate and some equivalent statements. Introduction to the Euclidean geometry (theorems for parallelograms, Intercept theorem and its relatives, similar triangles). Euclidean plane isometries: three mirrors suffice, the classification theorem. The classification of the Euclidean space isometries. Similarities, the fixpoint theorem and the classification of plane/space similarities. The general notion of congruence and similarity. Geometric measure theory: area of polygons, Jordan measure, the area of a circle. The axioms of measuring volumes, the volume of a sphere . The perimeter of a circle, the area of a sphere.

## Literature

## Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, <u>http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098\_college\_geometry/index.html</u> John Roe: Elementary Geometry, Oxford University Press, 1993.

#### Schedule:

 $l^{st}$  week Incidence axioms.  $2^{nd}$  week Ruler postulate, plane separation postulate.  $3^{rd}$  week Protractor postulate and the axiom of congruence.  $4^{th}$  week Some representative results in Absolute Geometry: congruence theorems.

Some representative results in Absolute Geometry: perpendicular and parallel lines, sufficient conditions for parallelism.

6<sup>th</sup> week

Inequalities.

7<sup>th</sup> week

The Euclidean parallel postulate and some equivalent statements.

8<sup>th</sup> week

Introduction to the Euclidean geometry (theorems for parallelograms, Intercept theorem and its relatives, similar triangles).

9<sup>th</sup> week

Euclidean plane isometries: three mirrors suffice, the classification theorem.

10<sup>th</sup> week

The classification of the Euclidean space isometries.

11<sup>th</sup> week

Similarities, the fixpoint theorem and the classification of plane/space similarities. The general notion of congruence and similarity.

12<sup>th</sup> week

Geometric measure theory: area of polygons, Jordan measure, the area of a circle.

13<sup>th</sup> week

The axioms of measuring volumes, the volume of a sphere.

14<sup>th</sup> week

The perimeter of a circle, the area of a sphere.

#### **Requirements:**

- for a signature

- for a grade

Attendance at **lectures** is recommended, but not compulsory. The course ends in an **examination**. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table:

Percent	Grade
0-60	fail (1)
61-70	pass (2)
71-80	satisfactory (3)
81-90	good (4)
91-100	excellent (5)

-an offered grade:

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

Title of course: Geometry 1. Code: TTMBG0301	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture:	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture:	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
Year, semester: 1st year, 1st semester	
Its prerequisite(s): -	

Further courses built on it: Geometry 2.

#### **Topics of course**

Triangles and circles. Trigonometry and its applications (inaccessible distances, visibility angles). Coordinate geometry and its applications (triangles and circles), intersections. Ruler-and-compass constructions. Inversive geometry, Mohr-Masceroni's theorem. The problem of Apollonius. Conics and the reflective properties (tangent lines to ellipses, paraboles and hyperboles). Rulerand-compass constructions related to conics. The geometry of the space (area, volume), revolution surfaces. Conic sections. The sphere (longitude and latitude), mappings of the sphere to the plane.

## Literature

Compulsory/Recommended Readings: Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098 college geometry/index.html John Roe: Elementary Geometry, Oxford University Press, 1993.

#### Schedule:

1<sup>st</sup> week Triangles. 2<sup>nd</sup> week Circles. 3<sup>rd</sup> week Trigonometry and its applications (inaccessible distances, visibility angles). 4<sup>th</sup> week Coordinate geometry and its applications (triangles). 5<sup>th</sup> week Coordinate geometry and its applications (circles). 6<sup>th</sup> week

Intersections.

7<sup>th</sup> week

Ruler-and-compass constructions.

8<sup>th</sup> week

Inversive geometry.

9<sup>th</sup> week

Mohr-Masceroni's theorem.

 $10^{th}$  week

The problem of Apollonius.

11<sup>th</sup> week

Conics and the reflective properties (tangent lines to ellipses, paraboles and hyperboles). Rulerand-compass constructions.

12<sup>th</sup> week

The geometry of the space (area, volume).

13<sup>th</sup> week

Revolution surfaces. Conic sections.

14<sup>th</sup> week

The sphere (longitude and latitude), mappings of the sphere to the plane.

## **Requirements:**

- for a signature

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

## - for a grade

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

Percent	Grade
0-60	fail (1)
61-70	pass (2)
71-80	satisfactory (3)
81-90	good (4)
91-100	excellent (5)

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

-an offered grade: -

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

<b>Title of course</b> : Geometry 2. <b>Code</b> : TTMBE0302	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 1st year, 2nd semester	
Its prerequisite(s): TTMBE0102, TTMBG0302 (p)	
Further courses built on it: Differential geometry	

Euclidean-Affin Geometry: vectors. Affine transformations, translations and central similarities. The ratio of three collinear points. Some representative results in Affine Geometry: Ceva's theorem, Menelaus' theorem. Analytic Euclidean-Affine geometry. Linear transformations, the general linear group. The analytic description of affine transformations. The fundamental theorem. Dot and cross product, vector triple product: the geometric characterization and the analytic formulas. Higher dimensional analytic geometry: reflections and isometries of the n-dimensional Euclidean space. The orthogonal group. Lower dimensional cases: two- and three-dimesnional spaces. Coordinate geometry: lines and planes. Implicite and parametric forms. Quadratic curves and surfaces. An introduction to Convex Geometry: convex sets and convex hulls. Carathéodory's theorem. Radon's lemma and Helly's theorem. Convex polygons and polyhedra. Euler's theorem, Descartes' theorem, regular convex polyhedra.

#### Literature

Compulsory/Recommended Readings:

S. R. Lay: Convex Sets and Their Applications, John Wiley & Sons, Inc., 1982. John Roe: Elementary Geometry, Oxford University Press, 1993. Csaba Vincze: Convex Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0025, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011\_0025\_mat\_14/index.html

#### Schedule:

1<sup>st</sup> week

Euclidean-Affin Geometry: vectors.

2<sup>nd</sup> week

Affine transformations, translations and central similarities.

 $3^{rd}$  week

The ratio of three collinear points. Some representative results in Affine Geometry: Ceva's

theorem, Menelaus' theorem. 4<sup>th</sup> week Analytic Euclidean-Affine geometry. Linear transformations, the general linear group. 5<sup>th</sup> week The analytic description of affine transformations. The fundamental theorem. 6<sup>th</sup> week Dot and cross product: the geometric characterization and the analytic formulas. 7<sup>th</sup> week Vector triple product: the geometric characterization and the analytic formula. 8<sup>th</sup> week Higher dimensional analytic geometry: reflections and isometries of the n-dimensional Euclidean space. 9<sup>th</sup> week The orthogonal group. 10<sup>th</sup> week Lower dimensional cases: two- and three-dimesnional spaces. 11<sup>th</sup> week Coordinate geometry: lines and planes. Implicite and parametric forms. 12<sup>th</sup> week Quadratic curves and surfaces. 13<sup>th</sup> week An introduction to Convex Geometry: convex sets and convex hulls. Carathéodory's theorem. Radon's lemma and Helly's theorem. 14<sup>th</sup> week Convex polygons and polyhedra. Euler's theorem, Descartes' theorem, regular convex polyhedra **Requirements:** - for a signature - for a grade Attendance at lectures is recommended, but not compulsory. The course ends in an examination. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table: Percent Grade 0-60 fail (1) pass(2)61-70 71-80 satisfactory (3) 81-90 good(4)91-100 excellent (5) -an offered grade:

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

Title of course: Geometry 2. Code: TTMBG0302		ECTS Credit points: 2	
Type of teaching, contact hours			
- lecture:			
- practice: 2 hours/week			
- laboratory: -			
Evaluation: mid-semester grade			
Workload (estimated), divided into contact hours:			
- lecture: 28 hours			
- practice: -			
- laboratory: -			
- home assignment: -			
- preparation for the exam: 32 hours			
Total: 60 hours			
Year, semester: 1st year, 2nd semester			
Its prerequisite(s): TTMEG0301			
Further courses built on it: Differential geometry			

The solution of geometric problems by vector algebra. The barycenter of a triangle and a tetrahedron. Linear dependency and independency, basis, coordinates. The simple ratio. The ellipse as the affine image of a circle. The area of an ellipse, compass-and-ruler constructions and the coordinate geometry of conics. Scalar, vector and mixed products. Computations in higher dimensional Euclidean vector spaces. Hyperplanes and reflections (analytic description). Reflections about lines and planes, rotations around lines and points. Lines, circles, planes and spheres. Intersections, distance and angles. Curves and surfaces (ruled- and revolution surfaces) implicit forms and parametrizations. Rolling without slipping: the cycloid. Twisted surfaces. Convex geometry.

#### Literature

Compulsory/Recommended Readings:

Csaba Vincze and László Kozma: College Geometry, TÁMOP-4.1.2.A/1-11/1-2011-0098, http://www.tankonyvtar.hu/hu/tartalom/tamop412A/2011-0098\_college\_geometry/index.html John Roe: Elementary Geometry, Oxford University Press, 1993. Vincze Csaba: Convex Geometry, University of Debrecen, 2013, TÁMOP-4.1.2.A/1-11/1-2011-0025.

## Schedule:

1<sup>st</sup> week

The solution of geometric problems by vector algebra.

 $2^{nd}$  week

The barycenter of a triangle and a tetrahedron.

3<sup>rd</sup> week

Linear dependency and independency, basis, coordinates.

4<sup>th</sup> week

The simple ratio.  $5^{th}$  week

3<sup>th</sup> week

The ellipse as the affine image of a circle. The area of an ellipse.

6<sup>th</sup> week

Compass-and-ruler constructions.

 $7^{th}$  week

The coordinate geometry of conics.

 $8^{th}$  week

Scalar, vector and mixed products.

9<sup>th</sup> week

Computations in higher dimensional Euclidean vector spaces. Hyperplanes and reflections (analytic description).

10<sup>th</sup> week

Reflections about lines and planes, rotations around lines and points

11<sup>th</sup> week

Lines, circles, planes and spheres. Intersections, distance and angles.

12<sup>th</sup> week

Curves and surfaces (ruled- and revolution surfaces) implicit forms and parametrizations.

13<sup>th</sup> week

Rolling without slipping: the cycloid. Twisted surfaces.

14<sup>th</sup> week

Convex geometry.

## **Requirements:**

## - for a signature

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

- for a grade

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

Percent	Grade
0-60	fail (1)
61-70	pass(2)
71-80	satisfactory (3)
81-90	good (4)
91-100	excellent (5)

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

-an offered grade: -

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

<b>Title of course</b> : Differential geometry <b>Code</b> : TTMBE0303	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: 22 hours	
- preparation for the exam: 40 hours	
Total: 90 hours	
Year, semester: 3 <sup>rd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBE0302, TTMBE0216	
Further courses built on it: -	

Introduction to the basic concepts, methods and theories of classical differential geometry: A curve in the 2- or 3-dimensional Euclidean plane space. Curvature and torsion. Surfaces in the Euclidean space. The first and second fundamental form of parameterized surfaces. Measurement on a surface. Curvature of a surface. Relationships between basic fundamental quantities. Parallelism on a surface. Geodesics. Variation of arc length. The minimizing properties of geodesics.

#### Literature

Compulsory: -

Recommended:

M. do Carmo: Differential Geometry of curves and Surfaces,

M. Spivak: A Comprehensive Introduction to Differential Geometry, Vol. 3,

B. O'Neill: Elementary Differential Geometry

#### Schedule:

1<sup>st</sup> week

A regular smooth curve in the Euclidean space. Parametrization and reparametrization of curves. Arc-length. Natural parametrization.

2<sup>nd</sup> week

Curvature of regular planar curves. Frenet base. Winding number of closed curves. Fundamental theorem of planar curves.

3<sup>rd</sup> week

Biregular curves in the 3-dimensional Euclidean space. Frenet-basis, curvature and torsion of a biregular curve.

4<sup>th</sup> week

Cartan matrix associated to a biregular curve. Fundamental theorem of curves in the 3-dimensional Euclidean space.

Surfaces in the Euclidean space. Parametrization of surfaces. Tangent space, normal direction at a point.

6<sup>th</sup> week

Measurement on surfaces. The first fundamental form of parametrized elemental surfaces. The arc length and angle of curves on surfaces. Surface area.

7<sup>th</sup> week

The second fundamental form of parametrized surfaces. Osculating paraboloids, Dupin indicatrix, classification of points on surfaces.

8<sup>th</sup> week

Gauss and Weingarten maps. Curvature curves on surfaces. Normal curvature. Meusnier's theorem.

9<sup>th</sup> week

Main or principal curvatures and directions. Rodriguez's theorem, Euler's formula. Product curvature and average curvature.

10<sup>th</sup> week

The Gauss basis associated to the parametrized surfaces. Christoffel symbols. Relationships between basic fundamental quantities. Theorema egregium. Bonnet's theorem.

11<sup>th</sup> week

Parallelism on a surface. Parallel translation along a curve. Geodesic curves on a surface. Geodesic curvature.

12<sup>th</sup> week

Variation problem of arc length. The minimizing properties of geodesics.

13<sup>th</sup> week

The Gauss-Bonnet theorem.

14<sup>th</sup> week

Surfaces with constant curvature.

## **Requirements:**

Only students who have signature from the practical part can take part of the exam. The exam is written. The grade is given according to the following table:

•	-	
Score		Grade
0-49		fail (1)
50-62		pass $(2)$
63-74		satisfactory (3)
75-86		good (4)
87-100		excellent (5)

Person responsible for course: Prof. Dr. Zoltán Muzsnay, university professor, DSc

Lecturer: Prof. Dr. Zoltán Muzsnay, university professor, DSc

Title of course: Differential geometry         Code: TTMBG0303	ECTS Credit points: 2	
Type of teaching, contact hours		
- lecture: -		
- practice: 2 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 42 hours		
- laboratory: -		
- home assignment: 18 hours		
- preparation for the exam:		
Total: 60 hours		
Year, semester: 3 <sup>rd</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): TTMBE0302, TTMBE0216		
Further courses built on it: -		
Topics of course		
Introduction to the basic concepts, methods and theories of classical differential geometry: A curve in the 2- or 3-dimensional Euclidean plane space. Curvature and torsion. Surfaces in the Euclidean space. The first and second fundamental form of parameterized surfaces. Measurement on a surface. Curvature of a surface. Relationships between basic fundamental quantities. Parallelism on a surface. Geodesics. Variation of arc length. The minimizing properties of geodesics.		
Literature		
Compulsory: -		
Recommended:		
M. do Carmo: Differential Geometry of curves and Surfaces, M. Snivak: A Comprehensive Introduction to Differential Geometry, Vol. 3		
B. O'Neill: Elementary Differential Geometry		
Schedule:		

A regular smooth curve in the Euclidean space. Parametrization and reparametrization of curves. Arc-length. Natural parametrization.

 $2^{nd}$  week

Curvature of regular planar curves. Frenet base. Winding number of closed curves. Fundamental theorem of planar curves.

 $3^{rd}$  week

Biregular curves in the 3-dimensional Euclidean space. Frenet-basis, curvature and torsion of a biregular curve.

4<sup>th</sup> week

Cartan matrix associated to a biregular curve. Fundamental theorem of curves in the 3-dimensional Euclidean space.

5<sup>th</sup> week

Surfaces in the Euclidean space. Parametrization of surfaces. Tangent space, normal direction at a point.

6<sup>th</sup> week

Measurement on surfaces. The first fundamental form of parametrized elemental surfaces. The arc length and angle of curves on surfaces. Surface area.

 $7^{th}$  week

The second fundamental form of parametrized surfaces. Osculating paraboloids, Dupin indicatrix, classification of points on surfaces.

 $8^{th}$  week

Test. Gauss and Weingarten maps. Curvature curves on surfaces. Normal curvature. Meusnier's theorem.

9<sup>th</sup> week

Main or principal curvatures and directions. Rodriguez's theorem, Euler's formula. Product curvature and average curvature.

10<sup>th</sup> week

The Gauss basis associated to the parametrized surfaces. Christoffel symbols. Relationships between basic fundamental quantities. Theorema egregium. Bonnet's theorem.

11<sup>th</sup> week

Parallelism on a surface. Parallel translation along a curve. Geodesic curves on a surface. Geodesic curvature.

12<sup>th</sup> week

Variation problem of arc length. The minimizing properties of geodesics.

13<sup>th</sup> week

Surfaces with constant curvature.

14<sup>th</sup> week

Test

## **Requirements:**

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

- for a grade

During the semester one test is written. The grade is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-84	good (4)
85-100	excellent (5)

Students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

## Person responsible for course: Prof. Dr. Zoltán Muzsnay, university professor, DSc

Lecturer: Prof. Dr. Zoltán Muzsnay, university professor, DSc

Title of course: Vector Analysis Code: TTMBE0304	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 3rd year, 2nd semester	
Its prerequisite(s): TTMBE0216, TTMBG0304(p)	
Further courses built on it: -	

Scalar fields: level curves and surfaces. The gradient and its geometric interpretation. Vector fields, the invariants of the Jacobian matrix: divergence and rotation (the vector invariant of the skew-symmetric part of the Jacobian). The Laplace operator. Parametrized curves, line integrals and work done. Stokes' theorem and its applications in the plane: conservative vector fields and potential (path independence for line integrals, rotation-free vector fields, exact differential equations). Parametrized surfaces, surface integrals: the flux. Gauss-Ostrogradsky theorem and Stokes' theorem in the space. Divergence and flux density. Rotation and circulation density. Identities and computational rules for vector operators: gradient, divergence and rotation. The derivative of the determinant function: the special linear group and its Lie algebra. The orthogonal group and its Lie algebra. Displacement fields: strain and rotational tensors. Integral curves and flows. Divergence-free vector fields (Liouville theorem, incompressible flows). Harmonic, subharmonic and superharmonic functions, the maximum principle.

#### Literature

Compulsory/Recommended Readings:

M. H. Protter, H. F. Weinberger: Maximum Principles in Differential Equations, Springer New York, 1984.

E. C. Young: Vector and Tensor Analysis, New York : M. Dekker, 1978.

#### Schedule:

1<sup>st</sup> week

Scalar fields: level curves and surfaces. The gradient and its geometric interpretation.

2<sup>nd</sup> week

Vector fields, the invariants of the Jacobian matrix: divergence and rotation (the vector invariant of the skew-symmetric part of the Jacobian). The Laplace operator.

3<sup>rd</sup> week

Parametrized curves, line integrals and work done.

4<sup>th</sup> week Stokes' theorem and its applications in the plane: conservative vector fields and potential (path independence for line integrals, rotation-free vector fields, exact differential equations). 5<sup>th</sup> week Parametrized surfaces, surface integrals: the flux. 6<sup>th</sup> week Gauss-Ostrogradsky theorem. 7<sup>th</sup> week Stokes' theorem in the space. 8<sup>th</sup> week Divergence and flux density. Rotation and circulation density. 9<sup>th</sup> week Identities and computational rules for vector operators: gradient, divergence and rotation. 10<sup>th</sup> week The derivative of the determinant function: the special linear group and its Lie algebra. 11<sup>th</sup> week The orthogonal group and its Lie algebra. 12<sup>th</sup> week Displacement fields: strain and rotational tensors. 13<sup>th</sup> week Integral curves and flows. Divergence-free vector fields (Liouville theorem, incompressible flows). 14<sup>th</sup> week Harmonic, subharmonic and superharmonic functions, the maximum principle. **Requirements:** - for a signature - for a grade Attendance at lectures is recommended, but not compulsory. The course ends in an examination. A practice grade at least 2 (pass) is a prerequisite to take an exam. The minimum requirement of the exam is more than 60 %. The grade is given according to the following table: Percent Grade 0-60 fail (1) 61-70 pass(2)satisfactory (3) 71-80 81-90 good(4)91-100 excellent (5)

-an offered grade:

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

Title of course: Vector analysis Code: TTMBG0302	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture:	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
Year, semester: 3rd year, 2nd semester	
Its prerequisite(s): TTMBE0216, TTMBG0304(p)	
Further courses built on it: -	

Scalar fields. Gradient and its geometric interpretation (level sets). Vector fields. Divergence and rotation. Laplacian. Identities. Parameterized curves. Line integrals. Stokes theorem in the plane and its applications: conservativity, potential, exact differential equations. Paremeterized surfaces. Surface integrals. Gauss-Ostrogradsky theorem and its consequences. Stokes theorem in the space and its applications. Newton's law of gravitation and its consequences: the conservativity of the gravitational field. Kepler's laws. Coordinate transforms and Jacobian (polar coordinates in the plane and in the space, cylindrical coordinates). The special linear group. The orthogonal group and its tangent space at the identity. Vector fields, integral curves and flows. Applications in the theory of differential equations. The maximum principle and its applications.

#### Literature

M. H. Protter, H. F. Weinberger: Maximum Principles in Differential Equations, Springer New York, 1984.

E. C. Young: Vector and tensor analysis, New York : M. Dekker, 1978.

#### Schedule:

 $l^{st}$  week Scalar fields and the gradient.  $2^{nd}$  week Vector fields. Divergence and rotation. Laplacian.  $3^{rd}$  week Identities and computational rules.  $4^{th}$  week The parameterization of curves. Line integrals.  $5^{th}$  week Stokes theorem in the plane and its applications: conservativity, potential, exact differential equations.

6<sup>th</sup> week

The parametrization of surfaces. Surface integrals.

7<sup>th</sup> week

Gauss-Ostrogradsky theorem and Stokes theorem in the space.

8<sup>th</sup> week

Newton's law of gravitation and its consequences: the conservativity of the gravitational field.  $g^{th}$  week

Kepler's laws.

10<sup>th</sup> week

Coordinate transforms and Jacobian (polar coordinates in the plane and in the space, cylindrical coordinates ).

11<sup>th</sup> week

The special linear group. The orthogonal group and its tangent space at the identity.

12<sup>th</sup> week

Vector fields, integral curves and flows.

13<sup>th</sup> week

Applications in the theory of differential equations.

14<sup>th</sup> week

The maximum principle. Harmonic-, subharmonic and superharmonic functions.

## **Requirements:**

## - for a signature

Participation at practice classes is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

## - for a grade

During the semester there are two tests. The minimum requirement for a grade is to get more than 60% of the total score of the two tests, separately. The grade is given according to the following table:

Percent	Grade
0-60	fail (1)
61-70	pass (2)
71-80	satisfactory (3)
81-90	good (4)
91-100	excellent (5)

If the score of any test is below 60%, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS. The final grade is the average of the grades of the two tests by the usual rules of rounding.

-an offered grade: -

Person responsible for course: Prof. Dr. Csaba Vincze, university professor, PhD

Lecturer: Prof. Dr. Csaba Vincze, university professor, PhD

<b>Title of course</b> : Measure and integral theory <b>Code</b> : TTMBE0205	ECTS Credit points: 3	
Type of teaching, contact hours		
- lecture: 2 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 28 hours		
- practice: -		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 62 hours		
Total: 90 hours		
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester		
Its prerequisite(s): TTMBE0215		
Further courses built on it: TTMBE0401, TTMBG0401		

Measure spaces and measures, their properties. Outer measures, pre-measures. Construction of measures. Lebesgue measure and its topological properties. Borel sets. The structure theorem of open sets. Approximation theorem. The properties of the Cantor set. Existence of non Lebesgue measurable sets. The Lebesgue–Stieltjes measure. Measurable functions and their basic properties, Lusin's theorem. Sequences of measurable functions. heorems of Lebesgue and Egoroff, Riesz's theorem on convergence in measure, approximation lemma. The Lebesgue integral of non-negative measurable functions. Beppo Levi's theorem, Fatou's lemma. The relation between the integral and the sum. Integrable functions. Lebesgue's majorized convergence theorem. The  $\sigma$ -additivity and the absolute continuity of the integral. The Lebesgue integral of complex functions. Lp spaces. Minkowski and Hölder inequality. The Riesz–Fischer theorem. The relation between the Riemann and the Lebesgue integral. Fubini's theorem. The n-dimensional Lebesgue measure. Lebesgue's differentiability theorem. Functions of bounded variation and absolute continuous functions. Basic properties of antiderivatives. The Newton–Leibniz formula.

#### Literature

Compulsory:

- H. Federer (1969): Geometric Measure Theory, Springer-Verlag

- Paul R. Halmos (1950): Measure Theory, D. Van Nostrand Company, Inc.

Recommended:

- Anthony W. Knapp (2005): Basic Real Analysis, Birkhauser

#### Schedule:

1<sup>st</sup> week

The definition of measure spaces and measures, and their most important properties.

#### 2<sup>nd</sup> week

Outer measures and their characterization, the notion of premeasures. Construction of measures.

The definition of the Lebesgue measure.

## 3<sup>rd</sup> week

The notion of the Lebesgue measure and its most important topological properties. Borel sets. The structure theorem of open sets. Approximation theorem.

## 4<sup>th</sup> week

The construction and most important properties of the Cantor set. Existence of not Lebesgue measurable sets.

## 5<sup>th</sup> week

Lebesgue-Stieltjes measure. The definition and fundamental properties of measurable functions, Luzin theorem.

## $6^{th}$ week

Sequences of measurable functions. The definition of convergence in measure and results related to it: theorems of Lebesgue and Egoroff, the selection theorem of Riesz, approximation lemma.

## $7^{th}$ week

The Lebesgue integral of nonnegative measurable functions and its basic properties. The theorem of Beppo Levi. Fatou lemma.

## $8^{th}$ week

The relation between the integral and the sum. Integrable functions and their fundamental properties.

## 9<sup>th</sup> week

Lebesgue's majorized convergence theorem. The  $\sigma\mbox{-}additivity$  and the absolute continuity of the integral.

## 10<sup>th</sup> week

The Lebesgue integral of complex functions. Lp spaces. Minkowski and Hölder inequality. The Riesz–Fischer theorem.

## 11<sup>th</sup> week

The relation between the Riemann and the Lebesgue integral. Fubini's theorem. The n-dimensional Lebesgue measure.

# 12<sup>th</sup> week

Lebesgue's differentiability theorem.

#### 13<sup>th</sup> week

Functions of bounded variation and absolute continuous functions. Basic properties of antiderivatives.

14<sup>th</sup> week

The Newton-Leibniz formula.

## **Requirements:**

The course ends in an **oral examination**. The process of the exam is as follows. First, a topic out of cca. eight is chosen randomly. The list of possible topics is made available for the students before the exam period. Then, the chosen topic should be elaborated in writing. Based partly on what has been written, an oral discussion of the topic follows, which also contains a few questions about other topics. The performance of the student during the exam is evaluated by a grade.

Attendance of lectures is recommended, but not obligatory.

Person responsible for course: Dr. Gergő Nagy, associate professor, PhD

Lecturer: Dr. Gergő Nagy, associate professor, PhD

Title of course: Probability theory         Code: TTMBE0401	ECTS Credit points: 4	
Type of teaching, contact hours		
- lecture: 3 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 42 hours		
- practice: -		
- laboratory: -		
- home assignment: 40 hours		
- preparation for the exam: 38 hours		
Total: 120 hours		
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): TTMBE0205		
Further courses built on it: -		
Topics of course		
Probability, random variables, distributions. Asymptotic theorems of probability theory.		
Literature		
<ul> <li><i>Compulsory:</i></li> <li>A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.</li> <li>Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972.</li> <li>Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.</li> </ul>		
Schedule:		
I <sup>st</sup> week		
Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.		
2 <sup>na</sup> week		
Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.		
Total probability theorem, the Bayes rule. Discrete random vari deviation. Binomial, hypergeometric, and Poisson distributions.	ables. Expectation, Standard	
4 <sup>th</sup> week		
Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution. $5^{th}$ week		

Expectation, variance and median. Uniform, exponential, normal distributions.

6<sup>th</sup> week

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.

 $8^{th}$  week

Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chisquared, Student's t, F-distributions.

9<sup>th</sup> week

Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.

 $10^{th}$  week

Kolmogorov's inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.

11<sup>th</sup> week

Characteristic function and its properties. Inversion formulas. Continuity theorem  $12^{th}$  week

Central limit theorem. Law of the iterated logarithm. Arcsine laws.

13<sup>th</sup> week

Conditional distribution function, conditional density function, conditional expectation.

14<sup>th</sup> week

Comparison of the limit theorems.

## **Requirements:**

- for a grade

Person responsible for course: Prof. Dr. István Fazekas, university professor, DSc

Lecturer: Prof. Dr. István Fazekas, university professor, DSc

<b>Title of course</b> : Probability theory <b>Code</b> : TTMBG0401	ECTS Credit points: 2	
Type of teaching, contact hours		
- lecture: -		
- practice: 2 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 28 hours		
- laboratory: -		
- home assignment: 16 hours		
- preparation for the exam: 16 hours		
Total: 60 hours		
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): TTMBE0205		
Further courses built on it: -		
Topics of course		
Probability, random variables, distributions. Asymptotic theorems of probability theory.		
Literature		
Compulsory:		
- A. N. Shiryayev: Probability, Springer-Verlag, Berlin, 1984.	V 1 I 1 1070	
- Ash, R. B.: Real Analysis and Probability. Academic Press, New York-London, 1972. - Bauer, H.: Probability Theory. Walter de Gruyter, Berlin-New York. 1996.		
Schedule:		
I <sup>st</sup> week		
Statistical observations. Numerical and graphical characteristics of the sample. Relative frequency, events, probability. Classical probability. Finite probability space.		
2 <sup>nu</sup> week		
Kolmogorov's probability space. Properties of probability. Finite and countable probability spaces. Conditional probability, independence of events. Borel-Cantelli lemma.		
3 <sup>ra</sup> week		
Total probability theorem, the Bayes rule. Discrete random variables. Expectation, Standard deviation. Binomial, hypergeometric, and Poisson distributions.		
4 <sup>th</sup> week		
Random variables, distribution, cumulative distribution function. Absolutely continuous distribution, probability density function. The general notion of distribution.		
5 <sup>th</sup> week		
Expectation, variance and median. Uniform, exponential, normal distributions.		

Joint distribution function and joint probability density function of random variables. Independent random variables, correlation coefficient.

Multivariate distributions. Expectation vector and variance matrix of random a random vector. Independence of random variables.

 $8^{th}$  week

Multivariate normal distribution, concentration ellipsoid. Sample from normal distribution. Chisquared, Student's t, F-distributions.

9<sup>th</sup> week

Weak law of large numbers. Almost sure convergence, convergence in distribution, convergence in probability, Lp convergence.

 $10^{th}$  week

Kolmogorov's inequality. Kolmogorov's three-series theorem. Strong laws of large numbers.

11<sup>th</sup> week

Characteristic function and its properties. Inversion formulas. Continuity theorem  $12^{th}$  week

Central limit theorem. Law of the iterated logarithm. Arcsine laws.

13<sup>th</sup> week

Conditional distribution function, conditional density function, conditional expectation.

14<sup>th</sup> week

Comparison of the limit theorems.

## **Requirements:**

- for a grade

Person responsible for course: Prof. Dr. István Fazekas, university professor, DSc

Lecturer: Prof. Dr. István Fazekas, university professor, DSc

<b>Title of course</b> : Introduction to informatics <b>Code</b> : TTMBG0601	ECTS Credit points: 2	
Type of teaching, contact hours		
- lecture: -		
- practice: 3 hours/week		
- laboratory: -		
Evaluation: mid-semester grade		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 42 hours		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 18 hours		
Total: 60 hours		
<b>Year, semester</b> : 1 <sup>st</sup> year, 2 <sup>nd</sup> semester		
Its prerequisite(s): -		
Further courses built on it: -		
Topics of course		
An introduction to LaTeX, a document preparation system for high-quality typesetting. Typesetting of complex mathematical formulas in LaTeX. Presentation creation using the Beamer class. Writing a formal or business letter in LaTeX. Using the modernev class for typesetting curricula vitae. The memoire class, a tool to create BSc/MSc thesis. Introduction to SageMath, a computer algebra package. The Jupyter Notebook interface and the SageMathCloud. Basic tools, assignment, equality, and arithmetic. Boolean expressions, loops, lists and sets. Writing functions in SageMath.		
Literature		
Compulsory:		
<i>Recommended:</i> T. Oetiker: The Not So Short Introduction to LaTeX. Gregory Bard: SageMath for Undergraduates (http://www.gregoryba	rd.com/Sage.html)	
Schedule:		
1 <sup>st</sup> week		
Basic usage of LaTeX. MikTeX and TeXLive distributions. The TeXmaker editor.		
2 <sup>nd</sup> week		
Preparing LaTeX documents, basic mathematical formulas in LaTeX.		
3 <sup>°°</sup> week		
$\mathcal{A}^{th}$ weak		
<sup>4</sup> weeκ Presentation in LaTeX, the beamer package and its usage. Special LaTeX commands in presentations. 5 <sup>th</sup> week		

Definitions, theorems in LaTeX, the memoire package and its usage to prepare thesis. The bibtex

package.

6<sup>th</sup> week

The modernev package, curriculum vitae and formal letter in LaTeX.

 $7^{th}$  week

First test.

8<sup>th</sup> week

The SageMath computer algebra package, basic mathematical usage.

9<sup>th</sup> week

Functions related to the rings of integers, computing the gcd and the extended euclidean algorithm.  $10^{th}$  week

Polynomial rings in SageMath, rational functions and related commands.

11<sup>th</sup> week

Sets and lists in SageMath, basic operations, loops in lists and sets.

12<sup>th</sup> week

Trigonometric functions in SageMath, expanding and simplifying expressions.

13<sup>th</sup> week

Defining functions in SageMath, preparing plots. Solving special equations, systems of equations.  $14^{th}$  week

Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the <u>following table</u>:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 – 85	good (4)
86 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. -*an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Szabolcs Tengely, university professor, DSc

Lecturer: Prof. Dr. Szabolcs Tengely, university professor, DSc

Title of course: Programming languages           Code: TTMBG0602	ECTS Credit points: 2	
Type of teaching, contact hours - lecture: -	· · · ·	
- practice: 2 hours/week		
- laboratory: -		
Evaluation: practice		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 28 hours		
- laboratory: -		
- home assignment: 16 hours		
- preparation for the exam: 16 hours		
Total: 60 hours		
Year, semester: 1 <sup>st</sup> year, 2 <sup>nd</sup> semester		
Its prerequisite(s): -		
Further courses built on it: -		
Topics of course		
Brief introduction to programming, programming languages, and architecture in general. Sequential, conditional, and repeated execution, reuse. Values and types, expressions. Container data types and standard uses. Reading and writing files. Text processing with string methods and regular expressions. Object-oriented design in practice. Basics of networked programming, working with data over the internet. Fundamentals of using databases and visualisation of data. Complex programming exercises.		
Literature		
Compulsory:		
- <i>Recommended:</i> Charles Severance, Python for Everybody: Exploring Data in Py Allen B. Downey, Think Python, 2012	ython 3, 2016.	
Schedule: 1 <sup>st</sup> week		

General introduction to programming and programming languages. Simplified structure of programs: sequential, conditional, and repeated execution, reuse. Flowcharts, errors and debugging. About Python.

 $2^{nd}$  week

Values and types, standard types in Python, differences between classes and types. Variables: assignment, naming conventions, and aliasing. Expressions: numerical and Boolean operations, orders of operations, short-circuit evaluation of logical expressions.

3<sup>rd</sup> week

Blocks and indentation in Python. Conditional execution: single conditionals, alternative executions, chained and nested conditionals, try and except. Repeated execution: definite loops, the range type, indefinite loops, infinite loops, and loop controls.

4<sup>th</sup> week

Functions: function calls, arguments and parameters, built-in and user-defined functions, fruitful and void functions, modules.

 $5^{th}$  week

Classification of container data types: iterable, mutable, and ordered types. Strings, lists, tuples, sets, and dictionaries and their basic roles.

 $6^{th}$  week

Files: open and close, read and write, creating new files and directories. Parsing strings with string methods.

 $7^{th}$  week

Regular expressions as a formal language and as strings with standard and meta characters. Parsing strings with regular expression methods.

 $8^{th}$  week

Object-oriented design: goals, principles, and patterns. Instances and methods: accessor, mutator, and manager methods. Classes in Python.

9<sup>th</sup> week

Networked programming: a brief introduction to HTML, XML, and JSON. Retrieving and processing content over the internet.

 $10^{th}$  week

Networked programming continued.

11<sup>th</sup> week

Databases: a brief introduction to databases and SQL. Reading, writing, and processing data from databases.

12<sup>th</sup> week

Visualization of data.

13<sup>th</sup> week

Extensive programming class.

14<sup>th</sup> week

Final exam.

## **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

## - for a grade

The course is evaluated on the basis of one written test and a homework project submitted during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the test is possible.

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. András Bazsó, assistant professor, PhD

Lecturer: Dr. András Bazsó, assistant professor, PhD

Title of course: Algorithms         Code: TTMBE0606	ECTS Credit points: 3	
Type of teaching, contact hours		
- lecture: 2 hours/week		
- practice: -		
- laboratory: -		
Evaluation: exam		
Workload (estimated), divided into contact hours:		
- lecture: 28 hours		
- practice: -		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 62 hours		
Total: 90 hours		
Year, semester: 1 <sup>st</sup> year, 2 <sup>nd</sup> semester		
Its prerequisite(s): -		
Further courses built on it: -		
Topics of course		
Classification of programming languages. Multi-character symbols. Data types. Instruction types. Loops. Subprograms. The role of algorithms in computing. Functions, recursive functions. Probabilistic analysis. Randomized algorithms. Heap, heapsort. Quicksort. Sorting in linear time. Elementary data structures.		
Literature		
Compulsory: - Recommended: T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introduction Cambridge, 2009 (3rd ed.) István Juhász: Programming Languages., http://www.tankonyvtar.hu/en/tartalom/tamop425/0046_programmin	n to Algorithms, MIT Press, ng_languages/index.html	
Schedule: $I^{st}$ week Introduction, foundations. Classification of programming languages. $2^{nd}$ week		

Multi-character symbols, symbolic names, labels, comments, literals, (constants.) Data types (simple, composite and pointer types).

3<sup>rd</sup> week

Assignment statements, the empty statements, the GOTO statement, selection statements, conditional statements, case/switch statement.

4<sup>th</sup> week

Loop statements, conditional loops, count-controlled loops, enumeration-controlled loops, infinite loops, composite loops.

5<sup>th</sup> week
Subprograms, the call chain and recursion, secondary entry points, parameter evaluation and parameter passing, block, scope, compilation unit.

6<sup>th</sup> week

The role of algorithms in computing. Algorithms as a technology, insertion sort, analyzing algorithms, designing algorithms.

 $7^{th}$  week

Functions, recursive functions. Grow of functions, asymptotic notation, standard notations and common functions. Recurrences, the substitution method and the recursion-tree method for solving recurrences.

 $\delta^{th}$  week

The master method and proof of the master method.

9<sup>th</sup> week

Probabilistic analysis, the hiring problem, indicator random variables.

10<sup>th</sup> week

Randomized algorithms and further examples of probabilistic analysis.

11<sup>th</sup> week

Heap, heapsort, maintaining the heap property, building the heap, the heapsort algorithm, priority queues.

12<sup>th</sup> week

Quicksort, description of quicksort, performance of quicksort, a randomized version of quicksort, analysis of quicksort.

13<sup>th</sup> week

Sorting in linear time, lower bounds for sorting, counting sort, radix sort, bucket sort.

14<sup>th</sup> week

Elementary data structures, stacks and queues, linked lists, implementing pointers and objects, representing rooted trees.

# **Requirements:**

- for a signature

If the student fail the course TTMBG0606, then the signature is automatically denied.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

<b>Total Score (%)</b>	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86-100	excellent (5)

# -an offered grade:

It is possible to obtain an offered grade on the basis of two written test during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)

	86 - 100	excellent (5)	
Person responsi	i <b>ble for course:</b> Dr. Nóra Györk	ös-Varga, assistant professor, Ph	D
Lecturer: Dr. N	óra Györkös-Varga, assistant pro	ofessor, PhD	

Title of course: Algorithms         Code: TTMBG0606	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: practice	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 32 hours	
Total: 60 hours	
<b>Year, semester</b> : 1 <sup>st</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): -	
Further courses built on it: -	
Topics of course	
Classification of programming languages. Multi-character symbols. Cycles. Subprograms. The role of algorithms in computing. Fur Probabilistic analysis. Randomized algorithms. Heap, heapsort. Quie Elementary data structures.	Data types. Instruction types. actions, recursive functions. cksort. Sorting in linear time.
Literature	
Compulsory: - Recommended: T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein: Introductio Cambridge, 2009 (3rd ed.) István Juhász: Programming Languages., http://www.tankonyvtar.hu/en/tartalom/tamop425/0046 programmin	n to Algorithms, MIT Press, ng languages/index.html

# Schedule:

1<sup>st</sup> week

Presentation of procedural and object-oriented languages, emphasizing the main differences and presentation of structures, parts of methods.

 $2^{nd}$  week

Presentation of data types (simple, composite and special), emphasizing the main differences of the types static and dynamic. Using the simpler and known data types (array, chain, list, structure), their creation from simple types.

 $3^{rd}$  week

Description of main types of statements, the difference of selection statements. Representing conditional statements (if-else) and case/switch statement (if-else if, or switch); presentation of the differences of "if-else if" and "switch" in case/switch statement. Recapitulate and exercise of notations and logical foundations required for conditional statements.

4<sup>th</sup> week

Loop statements, conditional loops, count-controlled loops, enumeration-controlled loops, infinite loops, composite loops. Programming loops "while" and "do-while", investigation of effect of different initialization and termination conditions concerning certain problems.

 $5^{th}$  week

Functions, planning of methods, determination of return values. Connection, linking of functions and methods. Presentation of recursive functions through some examples (e.g. Fibonacci sequence). Simultaneous determination of different return values with indicators.

 $6^{th}$  week

The role of algorithms in computing. Algorithms as a technology, insertion sort, bubble sort, analyzing algorithms, designing algorithms. Presentation and examination of different types of the (above) sorts with reference to efficiency.

 $7^{th}$  week

Grow of functions, asymptotic notation, standard notations and common functions. Recurrences, the substitution method and the recursion-tree method for solving recurrences.

 $8^{th}$  week

The master method, practical importance of the master method.

9<sup>th</sup> week

Probabilistic analysis, the hiring problem, indicator random variables.

10<sup>th</sup> week

Randomized algorithms and further examples of probabilistic analysis.

11<sup>th</sup> week

Heap, heapsort, maintaining the heap property, building the heap, the heapsort algorithm, priority queues.

12<sup>th</sup> week

Quicksort, description of quicksort, performance of quicksort, a randomized version of quicksort, analysis of quicksort.

13<sup>th</sup> week

Sorting in linear time, lower bounds for sorting; programming the counting sort, radix sort, bucket sort. Presentation of foundation of Hash functions and using them for sort of certain array.

14<sup>th</sup> week

Elementary data structures, stacks and queues, linked lists, implementing pointers and objects, representing rooted trees.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the test is possible. *-an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Dr. Nóra Györkös-Varga, assistant professor, PhD

Lecturer: Dr. Nóra Györkös-Varga, assistant professor, PhD

<b>Title of course</b> : Applied number theory <b>Code</b> : TTMBE0109	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0106	
Further courses built on it: TTMBE0111	
Topics of course	

Basic notions of complexity theory. Some basic algorithms and their complexity. Approximation of real numbers by rationals, the theorem of Dirichlet. Liouville's theorem, a construction of transcendental numbers. Continued fractions and their properties. Finite and infinite continued fractions. Approximation with continued fractions. The LLL-algorithm and some of its applications. Pseudoprimes and their properties. Carmichael-numbers and their role in probabilistic prime tests. Euler-pseudoprimes and their properties. The Soloway-Strassen probabilistic prime test. Strong pseudoprimes and their properties. The Miller-Rabin probabilistic prime test. Deterministic prime tests, Wilson's theorem, the description of the Agrawal-Kayal-Saxena test. The birthday paradox and Pollard's p-method. Fermat-factorization. Factorization with a factor basis. Factorization with continued fractions.

# Literature

*Compulsory*:

Recommended:

Neal Koblitz: A Course in Number Theory and Cryptography, Springer Verlag, 1994.

I. Niven, H. S. Zuckerman, H. L. Montgomery: An Introduction to the Theory of Numbers, Wiley, 1991.

Nigel Smart: The Algorithmic Resolution of Diophantine Equations, London Mathematical Society Student Text 41, Cambridge University Press, 1998.

# Schedule:

1<sup>st</sup> week

Basic concepts of complexity theory. Some fundamental algorithms and their complexity. Solution of related problems.

 $2^{nd}$  week

Approximation of real numbers by rationals, Dirichlet's theorem. Solution of related problems. 3<sup>rd</sup> week

Liouville's theorem, construction of transcendental numbers. Solution of related problems.

4<sup>th</sup> week

Continued fractions and their properties. Finite and infinite continued fractions. Solution of related problems.

 $5^{th}$  week

Approximation by continued fractions. Solution of related problems.

 $6^{th}$  week

The LLL-algorithm and some of its applications. Solution of related problems.

 $7^{th}$  week

Pseudoprimes and their properties. Carmichael-numbers and their role in probabilistic prime tests. Solution of related problems.

 $8^{th}$  week

Euler-pseudoprimes and their properties. The Soloway-Strassen probabilistic prime test. Solution of related problems.

9<sup>th</sup> week

Strong pseudoprimes and their properties. The Miller-Rabin probabilistic prime test. Solution of related problems.

 $10^{th}$  week

Deterministic prime tests, Wilson's theorem, the Agrawal-Kayal-Saxena test. Solution of related problems.

11<sup>th</sup> week

The birthday paradox and Pollard's-  $\rho$ -method. Solution of related problems.

12<sup>th</sup> week

Fermat-factorization. The background of the method and its variants. Solution of related problems.  $13^{th}$  week

Factorization with a factor basis.. Solution of related problems.

14<sup>th</sup> week

Continued fraction factorization. The background of the method and its applications. Solution of related problems.

# **Requirements:**

- for a signature

Signature is not a basis of evaluation in this course.

- for a grade

The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 80	good (4)
81 - 100	excellent (5)

-an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Lajos Hajdu, university professor, DSc

Lecturer: Prof. Dr. Lajos Hajdu, university professor, DSc

<b>Title of course</b> : Algorithms in algebra and number theory <b>Code</b> : TTMBG0110	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: -	
- practice: 3 hours/week	
- laboratory: -	
Evaluation: practice	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 42 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 48 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0106	
Further courses built on it: -	
Topics of course	
Linear algebra and applications using SageMath. Factoring poly Berlekamp algorithm. Shamir's secret sharing algorithm. Latt applications. Number theoretic functions in SageMath. Linea Frobenious problem. Conics and elliptic curves in SageMath.	rnomials over finite fields, the tices, the LLL-algorithm and ar Diophantine equations, the
Literature	
Compulsory:	
-	

Recommended:

Victor Shoup: A Computational Introduction to Number Theory and Algebra, Cambridge University Press, 2005.

William Stein: Elementary Number Theory: Primes, Congruences, and Secrets, Springer-Verlag, 2008

# Schedule:

1<sup>st</sup> week

A short introduction of SageMath (basic structures, lists, sets, programming tools).

 $2^{nd}$  week

Linear algebra over the reals, complex numbers and finite fields. The Berlekamp algorithm. Computing formulas for recurrence sequences.

3<sup>rd</sup> week

Polynomials and matrices over finite fields, the Samir secret sharing procedure.

4<sup>th</sup> week

The NTRU cryptosystem and its implementation in SageMath.

5<sup>th</sup> week

Polinomial equations and applications.

6<sup>th</sup> week

Number theoretical functions in SageMath, linear Diophantine equations. Combinatorial 7<sup>th</sup> week

First test.

8<sup>th</sup> week

Lattices and the LLL-algorithm in SageMath. The knapsack problem.

9<sup>th</sup> week

The Frobenius problem. Solutions of the Frobenius problem via Wilf-method and Brauer-method.  $10^{th}$  week

Computations related to quadratic residues, the Legendre symbol.

11<sup>th</sup> week

The ternary Diophantine equation  $ax^{2}+by^{2}+cz^{2}=0$ . Descent algorithm to determine integral solutions, parametrization of rational and integral points.

12<sup>th</sup> week

Elliptic curves, points on elliptic curves over the rationals, finite fields. Applications of elliptic curves.

13<sup>th</sup> week

Points on elliptic curves over finite fields, determining the order, the baby step-giant step algorithm.

14<sup>th</sup> week

Second test.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of two written tests during the semester. The grade is given according to the <u>following table</u>:

Total Score (%)	Grade
0-50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71-85	good (4)
91 - 100	excellent (5)

If a student fail to pass at first attempt, then a retake of the tests is possible. *-an offered grade:* 

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Szabolcs Tengely, university professor, DSc

Lecturer: Prof. Dr. Szabolcs Tengely, university professor, DSc

<b>Title of course</b> : Introduction to cryptography <b>Code</b> : TTMBE0111	ECTS Credit points: 3
Type of teaching, contact hours <ul> <li>lecture: 2 hours/week</li> <li>practice: -</li> <li>laboratory: -</li> </ul>	
Evaluation: exam	
Workload (estimated), divided into contact hours: <ul> <li>lecture: 28 hours</li> <li>practice: -</li> <li>laboratory: -</li> <li>home assignment: -</li> <li>preparation for the exam: 62 hours</li> </ul> Total: 90 hours Year, semester: 3 <sup>rd</sup> year, 1 <sup>st</sup> semester Its prerequisite(s): TTMBE0109 Eurther courses built on it: -	
Topics of course	
Basic cryptographic concepts. Symmetric and asymmetric cryptos cryptosystems, DES, AES. The RSA cryptosystem and the analy logarithm problem. Algorithms for solving the discrete logarithm on the discrete logarithm problem. Elliptic curve cryptography. Digital signature. The basics of PGP.	systems. The Cesar and the linear ysis of its security. The discrete n problem. Cryptosystems based . Basic cryptographic protocols.
Literature	
Compulsory: -	

Recommended:

J. Buchmann: Einführung in die Kryptographie, Springer, 1999.

N. Koblitz: A Course in Number Theory and Cryptography, Springer, 1987.

#### Schedule:

*I<sup>st</sup> week* 

The models of information transfer. The Shannon model, modulator, demodulator and their parts. The two branches of cryptology: cryptography and cryptanalysis. Major applications of cryptography. Requirements of a modern cryptosystem. The notion of cryptosystem and its mathematical model.

 $2^{nd}$  week

Classical cryptosystems. Symmetric versus asymmetric cryptosystems. The Caesar cryptosystem, substitution ciphers, the affine cryptosystem. Frequency analysis. Breaking the classical cryptosystems.

 $3^{rd}$  week

Block-cyphers. Feistel-type ciphers. The history of the DES, requirements for a cryptosystem in those times. Description of the DES. Security of the DES. Double DES and triple DES.

 $4^{th}$  week

The field GF( $2^8$ ). Operations in GF( $2^8$ ). Bytes as elements of GF( $2^8$ ). The structure of the polynomial ring  $GF(2^8)[x]$  and of the factor ring  $GF(2^8)[x]/(x^4+1)$ , operations in the factor ring  $GF(2^8)[x]/(x^4+1)$ . 5<sup>th</sup> week The call for proposals for AES. Requirements concerning AES. The winner of the call: the Rijndael. Description of the Rijndael cryptosystem: number of rounds, round-transformation final round, round-key generation. 6<sup>th</sup> week The basic idea behind the public-key cryptosystems, the infrastructure of public key cryptosystems. The idea behind the RSA cryptosystem. Description of the RSA cipher. 7<sup>th</sup> week First test. 8<sup>th</sup> week The security of the RSA - correct choice of the parameters. Known protocol errors and possibilities of attacking the RSA in case of wrong parametrization or programming. 9<sup>th</sup> week The discrete logarithm problem. The Pohlig-Hellman algorithm, the Baby-step Giant-step algorithm, the Pollard-rho algorithm, and the Index-calculus algorithm. 10<sup>th</sup> week Public-key cryptosystems based on the hardness of the discrete logarithm problem: the El Gamal cryptosystem, the Diffie-Hellmann key-exchange protocol, the Massey-Omura cryptosystem. 11<sup>th</sup> week Definition of elliptic curves. Points on elliptic curves over a given field. Definition of the group of an elliptic curve. The real case. Elliptic curves over finite fields. Hasse's theorem. 12<sup>th</sup> week Encoding the plaintext as a point of an elliptic curve. Cryptosystems based on the discrete logarithm problem over elliptic curves: the El Gamal cryptosystem, the Massey-Omura cryptosystem. 13<sup>th</sup> week Protocols for key-exchange, digital signature and authentication. Zero-knowledge proofs. 14<sup>th</sup> week Second test. **Requirements:** - for a signature If the student fail the course TTMBG0111, then the signature is automatically denied. - for a grade The course ends in oral examination. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86 - 100	excellent (5)

-an offered grade:

It is possible to obtain an offered grade on the basis of two written test during the semester. The grade is given according to the following table:

	Total Score (%)	Grade
	0 - 50	fail (1)
	51 - 60	pass (2)
	61 - 70	satisfactory (3)
	71 - 85	good (4)
	86 - 100	excellent (5)
Person respons	ible for course: Prof. Dr. Attila	Bérczes, university professor, D
Lecturer: Prof.	Dr. Attila Bérczes, university p	rofessor, DSc

<b>Title of course</b> : Introduction to cryptography <b>Code</b> : TTMBG0111	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: 16 hours	
- preparation for the exam: 16 hours	
Total: 60 hours	
<b>Year, semester</b> : 3 <sup>rd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBE0109	
Further courses built on it: -	
Topics of course	
Basic cryptographic concepts. Symmetric and asymmetric crypto cryptosystems, DES, AES. The RSA cryptosystem and the anal logarithm problem. Algorithms for solving the discrete logarithm on the discrete logarithm problem. Elliptic curve cryptography Digital signature. The basics of PGP.	systems. The Cesar and the linear lysis of its security. The discrete m problem. Cryptosystems based 7. Basic cryptographic protocols.
Literature	
Compulsory:	
- Recommended:	
J. Buchmann: Einführung in die Kryptographie, Springer, 1999.	
N. Koblitz: A Course in Number Theory and Cryptography, Springer, 1987.	
Schedule:	
1 <sup>st</sup> week	
Introduction to the Magma computer algebra system.	
2 <sup>nd</sup> week	
Realization of the Caesar cryptosystem, a substitution cypher, or an affine cryptosystem.	
3 <sup>rd</sup> week	
Programming the DES in frame of group work.	
4 <sup>th</sup> week	
Continuing the implementation of DES. Combining the individu	ally produced programme-parts.
5 <sup>th</sup> week	

Computer aided computations in the field  $GF(2^8)$ , the polynomial ring  $GF(2^8)[x]$  and the factor ring  $GF(2^8)[x]/(x^4+1)$  using Magma.

 $6^{th}$  week

Group work: programming the encryption/decryption function of the Rijndael cryptosystem.  $7^{th}$  week

Continuing the implementation of the Rijndael cryptosystem. Combining the individually produced programme-parts.

 $8^{th}$  week

Implementing the RSA cryptosystem.

9<sup>th</sup> week

Programming one of the algorithms for solving the DLP.

10<sup>th</sup> week

Implementing one of the cryptosystems based on the hardness of the DLP.

11<sup>th</sup> week

Defining and manipulating elliptic curves in Magma.

12<sup>th</sup> week

Writing a programme to encode plaintext as a point on an elliptic curve.

13<sup>th</sup> week

Implementing the Diffie-Hellmann key-exchange protocol.

14<sup>th</sup> week

Evaluation, decision of the marks of the students.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

- for a grade

The course is evaluated on the basis of one written test and a homework project submitted during the semester. The grade is given according to the following table:

Total Score (%)	Grade
0 - 50	fail (1)
51 - 60	pass (2)
61 - 70	satisfactory (3)
71 - 85	good (4)
86-100	excellent (5)

If a student fail to pass at first attempt, then a retake of the test is possible. -an offered grade:

It is not possible to obtain an offered grade in this course.

Person responsible for course: Prof. Dr. Attila Bérczes, university professor, DSc

Lecturer: Prof. Dr. Attila Bérczes, university professor, DSc

Title of course: Numerical analysis	ECTS Credit points: 4
Code: TTMBE0209	
Type of teaching, contact hours	
- lecture: 3 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 42 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 78 hours	
Total: 120 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0102, TTMBE0215, TTMBG0209(p)	
Further courses built on it: -	

Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its variants: its algorithms, operational complexity, pivoting. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iterative methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximation. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

### Literature

Compulsory:

Recommended:

- Atkinson, K.E.: Elementary Numerical Analysis. John Wiley, New York, 1993.

- Lange, K.: Numerical analysis for statisticians. Springer, New York, 1999.

- Press, W.H. – Flannery, B.P. – Tenkolsky, S.A. – Vetterling, W.T.: Numerical recipes in C. Cambridge University Press, Cambridge, 1988.

- Engeln-Mullgens, G. - Uhling, F.: Numerical algorithms with C. Springer, Berlin, 1996.

#### Schedule:

 $l^{st}$  week Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems.

 $2^{nd}$  week Solution of system of linear equations: Gaussian elimination and its variants

3<sup>rd</sup> week Algorithms of the Gauss elimination and its operational comlexity. Pivoting.

4<sup>th</sup> week Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky factorisation, QR factorisation of matrices.

 $5^{th}$  week Iterative methods for solving linear systems: Gauss-Seidel iteration and its convergence  $6^{th}$  week Preconditioning. The gradient method and the conjugate gradient method

7<sup>th</sup> week Approximate solution of nonlinear equations: Newton method, local and global convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden-method

 $\delta^{th}$  week Numerical methods for solving eigenvalue problems: power method and inverse iteration  $9^{th}$  week Numerical methods for solving eigenvalue problems: shift method, the QR algorithm

 $10^{th}$  week Interpolation and approximation problems: Lagrange-interpolation, Hermite-interpolation. Spline interpolation. Error of the approximation. Tschebisev-approximation

11<sup>th</sup> week Numerical integration: Newton-Cotes formulas. Composite quadrature formulas

12<sup>th</sup> week Gauss quadrature. Existence, convergence, error estimation

*13<sup>th</sup> week* Numerical methods for solving initial value problems of ordinary differential equations: Euler method, Runge-Kutta method

14<sup>th</sup> week Numerical methods for solving boundary value problems of ordinary differential equations: finite difference methods, finite element method

## **Requirements:**

- for a grade

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-89	good (4)
90-100	excellent (5)

Person responsible for course: Dr. Borbála Fazekas, associate professor, PhD

Lecturer: Dr. Borbála Fazekas, associate professor, PhD

Title of course: Numerical analysis	ECTS Credit points: 2
Code: TTMBG0209	*
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: practical	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the tests: 32 hours	
Total: 60 hours	
Year, semester: 2 <sup>nd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0102, TTMBE0215	
Further courses built on it: -	

Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Gaussian elimination and its variants: its algorithms, operational complexity, pivoting. Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky decomposition, QR decomposition. Iterative methods for solving linear and nonlinear systems: Gauss-Seidel iteration, gradient method, conjugate gradient method, Newton method, local and global convergence, quasi-Newton method, levenberg–Marquardt algorithm, Broyden method. Solving eigenvalue problems: power method, inverse iteration, translation, QR method. Interpolation and approximation problems: Lagrange and Hermite interpolation, spline interpolation, Chebyshev-approximation. Quadrature rules: Newton-Cotes formulas, Gauss quadrature. Numerical methods for solving ordinary differential equations: Euler method, Runge-Kutta methods, finite-difference methods, finite element method.

# Literature

Compulsory:

Recommended:

- Atkinson, K.E.: Elementary Numerical Analysis. John Wiley, New York, 1993.

- Lange, K.: Numerical analysis for statisticians. Springer, New York, 1999.

- Press, W.H. – Flannery, B.P. – Tenkolsky, S.A. – Vetterling, W.T.: Numerical recipes in C. Cambridge University Press, Cambridge, 1988.

- Engeln-Mullgens, G. – Uhling, F.: Numerical algorithms with C. Springer, Berlin, 1996.

#### Schedule:

 $I^{st}$  week Features of computions by computer, error propagation. Some important matrix transformations for solving linear systems and eigenvalue problems. Solution of system of linear equations: Gaussian elimination and its variants

2<sup>nd</sup> week Algorithms of the Gauss elimination and its operational complexity. Pivoting.

*3<sup>rd</sup> week* Decompositions of matrices: Schur complement, LU decomposition, LDU decomposition, Cholesky factorisation, QR factorisation of matrices

4<sup>th</sup> week Iterative methods for solving linear systems: Gauss-Seidel iteration and its convergence

5<sup>th</sup> week Preconditioning. The gradient method and the conjugate gradient method

 $6^{th}$  week Approximate solution of nonlinear equations: Newton method, local and global

convergence, quasi-Newton method, Levenberg–Marquardt algorithm, Broyden-method 7<sup>th</sup> week Test

 $\delta^{th}$  week Numerical methods for solving eigenvalue problems: power method and inverse iteration. Shift method, the QR algorithm

9<sup>th</sup> week Interpolation and approximation problems: Lagrange-interpolation, Hermiteinterpolation. Spline interpolation. Error of the approximation. Tschebisev-approximation

10<sup>th</sup> week Numerical integration: Newton-Cotes formulas. Composite quadrature formulas

11th week Gauss quadrature. Existence, convergence, error estimation

*12<sup>th</sup> week* Numerical methods for solving initial value problems of ordinary differential equations: Euler method, Runge-Kutta method

*13<sup>th</sup> week* Numerical methods for solving boundary value problems of ordinary differential equations: finite difference methods, finite element method

14<sup>th</sup> week Test

#### **Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7<sup>th</sup> week and the other test in the 14<sup>th</sup> week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-89	good (4)
90-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

**Person responsible for course:** Dr. Borbála Fazekas, associate professor, PhD **Lecturer:** Dr. Borbála Fazekas, associate professor, PhD

<b>Title of course</b> : Economic mathematics <b>Code</b> : TTMBE0211	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 3 <sup>rd</sup> year, 1st semester	
Its prerequisite(s): TTMBE0211	
Further courses built on it-	

Computation of future and present values, discounted present value and investment projects. Bounds for the budget, change of the budget line, consumer preferences, preference order. Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility. Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity. Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity. Production functions, marginal rate of substitution. CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery– Minhas–Solow type production function. Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly. Individual and social preferences, social welfare function. Arrow's impossibility theorem. Consistent aggregation, bisymmetry equation. Influencing the distribution of incomes, the discounted present value of continuous income stream, Lorenz curve, Gini coefficient. Leontieff models.

#### Literature

#### Compulsory:

-

Recommended:

- M. Carter: Foundations of Mathematical Economics, MIT Press, 2001.

- K. Sydsaeter, P. Hammond, Mathematics for Economic Analysis, Pearson Publishing, 1995.
- H. R. Varian: Intermediate Microeconomics: A Modern Approach, W.W. Norton, 1987.

#### Schedule:

1<sup>st</sup> week

Computation of future and present values, discounted present value and investment projects.  $2^{nd}$  week

Bounds for the budget, change of the budget line, consumer preferences, preference order.

3<sup>rd</sup> week

Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility.

 $4^{th}$  week

Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity.

5<sup>th</sup> week

Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity.

 $6^{th}$  week

Production functions, marginal rate of substitution.

7<sup>th</sup> week

CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function.

 $8^{th}$  week

Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly.

9<sup>th</sup> week

Individual and social preferences, social welfare function.

10<sup>th</sup> week

Arrow's impossibility theorem.

11<sup>th</sup> week

Consistent aggregation, bisymmetry equation.

12<sup>th</sup> week

Influencing the distribution of incomes, the discounted present value of continuous income stream,  $13^{th}$  week

Lorenz curve, Gini coefficient.

14<sup>th</sup> week

Leontieff models.

#### **Requirements:**

Attendance at **lectures** is recommended, but not compulsory.

- for a grade

The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table:

Score	Grade
0-49	fail (1)
50-62	pass (2)
63-76	satisfactory (3)
77-88	good (4)
89-100	excellent (5)

Person responsible for course: Dr. Fruzsina Mészáros, assistant professor, PhD

Lecturer: Dr. Fruzsina Mészáros, assistant professor, PhD

<b>Title of course</b> : Economic mathematics <b>Code</b> : TTMBG0211		ECTS Credit points: 2
Type of teaching, contact hours		
- lecture: -		
- practice: 2 hours/week		
- laboratory: -	- laboratory: -	
Evaluation: practical		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 28 hours		
- laboratory: -		
- home assignment: -		
- preparation for the tests: 32 hours		
Total: 60 hours		
Year, semester: 3 <sup>rd</sup> year, 1st semester		
Its prerequisite(s): TTMBE0211		
Further courses built on it:-		

Computation of future and present values, discounted present value and investment projects. Bounds for the budget, change of the budget line, consumer preferences, preference order. Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility. Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity. Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity. Production functions, marginal rate of substitution. CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery– Minhas–Solow type production function. Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly. Individual and social preferences, social welfare function. Arrow's impossibility theorem. Consistent aggregation, bisymmetry equation. Influencing the distribution of incomes, the discounted present value of continuous income stream, Lorenz curve, Gini coefficient. Leontieff models.

### Literature

#### Compulsory:

Recommended:

- M. Carter: Foundations of Mathematical Economics, MIT Press, 2001.

- K. Sydsaeter, P. Hammond, Mathematics for Economic Analysis, Pearson Publishing, 1995.
- H. R. Varian: Intermediate Microeconomics: A Modern Approach, W.W. Norton, 1987.

## Schedule:

1<sup>st</sup> week

Computation of future and present values, discounted present value and investment projects.  $2^{nd}$  week

Bounds for the budget, change of the budget line, consumer preferences, preference order.  $3^{rd}$  week Indifference curves, marginal rate of substitution, utility, utility functions, Cobb-Douglas preferences, marginal utility.

4<sup>th</sup> week

Optimal choice, consumer demand, demand curves, inverse demand curve, market demand, elasticity.

 $5^{th}$  week

Demands of constant elasticity, elasticity and marginal revenue, marginal revenue curves, income elasticity.

 $6^{th}$  week

Production functions, marginal rate of substitution.

7<sup>th</sup> week

CES property, Cobb–Douglas type production function and its properties, Arrow–Chenery–Minhas–Solow type production function.

 $8^{th}$  week

Equilibrium points of two-person games, the best response mapping, game theoretic model of Bertrand and Cournot type duopoly.

9<sup>th</sup> week

Individual and social preferences, social welfare function.

10<sup>th</sup> week

Arrow's impossibility theorem.

11<sup>th</sup> week

Consistent aggregation, bisymmetry equation.

12<sup>th</sup> week

Influencing the distribution of incomes, the discounted present value of continuous income stream, 13<sup>th</sup> week

13<sup>th</sup> week

Lorenz curve, Gini coefficient.

14<sup>th</sup> week

Leontieff models.

# **Requirements:**

- for a practical

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behaviour or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7<sup>th</sup> week and the other test in the 14<sup>th</sup> week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

Score	Grade
0-49	$\operatorname{tail}(1)$
63-76	satisfactory (3)
77-88	good (4)

89-100

excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Fruzsina Mészáros, assistant professor, PhD

Lecturer: Dr. Fruzsina Mészáros, assistant professor, PhD

<b>Title of course</b> : Analysis with computer <b>Code</b> : TTMBG0210	ECTS Credit points: 4
Type of teaching, contact hours	I
- lecture: -	
- practice: -	
- laboratory: 3 hours/week	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: -	
- laboratory: 42 hours	
- home assignment: -	
- preparation for the test: 78 hours	
Total: 120 hours	
Year, semester: 3 <sup>rd</sup> year, 2 <sup>nd</sup> semester	
Its prerequisite(s): TTMBE0215	
Further courses built on it: -	

The Maple; types of data, simple for-cycles, defining functions. Examination of functions; continuity, limit, zeros, extrema, constrained extrema. Differentiation, integration and numerical integration. Programming of simple quadrature rules. Solving differential equations with analytic methods and visualizing the solutions. Solving differential equations with numerical methods, programming Runge–Kutta fomulas. Ways of defining vectors and matrces. Vector and matrix operations, decompositions of matrices. Solving linear systems of equations with direct and iterative methods. Graphic tools in two dimension: making figures, rotation, reflection, parametrized curves. Graphic tools in three dimension: functions of two variables, space curves, surfaces, solid figures. Making animations, illustrating geometric and physical problems. Curve fitting; Lagrange and Hermite interpolation, Bezier curves and spline interpolation. For-cycle and while-cycle, conditional branches. Writing simple procedures: searching for primes, recursive functions, divisibility problems. Writing complex procedures: numerical differentiation and integration, approximation of functions, orthogonal polynomials and differential equations.

# Literature

Compulsory: -

Recommended:

- W. Gander, J. Hrebicek: Solving Problems in Scientific Computing Using Maple and MATLAB. Springer-Verlag, Berlin, Heidelberg, New York, 1993, 1995.

## Schedule:

*I<sup>st</sup> week* Introduction. Data types of Maple: simple data types, complex data types.

 $2^{nd}$  week Simple for-cycles. Defining functions. Examination of functions; continuity, limit, zeros, extrema, constrained extrema.

 $3^{rd}$  week Differentiation, integration and numerical integration. Programming of simple quadrature rules.

*4<sup>th</sup> week* Solving differential equations with analytic methods and visualizing the solutions. Solving differential equations with numerical methods, programming simple Runge–Kutta fomulas.

5<sup>th</sup> week Ways of defining vectors and matrices. Vector and matrix operations, decompositions of matrices.

 $6^{th}$  week Solving linear systems of equations with direct and iterative methods. Programming of simple iterative methods.

 $7^{th}$  week Graphic tools in two dimension: making figures, rotation, reflection, parametrized curves.  $8^{th}$  week Graphic tools in three dimension: functions of two variables, space curves, surfaces, solid figures.

9<sup>th</sup> week Making animations, illustrating geometric and physical problems.

 $10^{th}$  week Curve fitting; Lagrange and Hermite interpolation, Bezier curves and spline interpolation.

11<sup>th</sup> week For-cycle and while-cycle, conditional branches.

12<sup>th</sup> week Writing simple procedures: sequences, Taylor-series, extrema of functions.

13<sup>th</sup> week Writing complex procedures: numerical differentiation and integration, approximation of functions, orthogonal polynomials and differential equations.

14<sup>th</sup> week Test

## **Requirements:**

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there is one test in the 14<sup>th</sup> week.

The minimum requirement for the test is 50%. Based on the score of the test, the grade for the test is given according to the following table:

Score	Grade
0-49	fail (1)
50-59	pass (2)
60-74	satisfactory (3)
75-89	good (4)
90-100	excellent (5)

If the score of the test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Borbála Fazekas, assistant professor, PhD

Lecturer: Dr. Borbála Fazekas, assistant professor, PhD

<b>Title of course</b> : Computer geometry <b>Code</b> : TTMBG0308	ECTS Credit points: 3
Type of teaching, contact hours	
- lecture: -	
- practice: 3 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 42 hours	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 48 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 1 <sup>st</sup> semester	
Its prerequisite(s): TTMBE0302	
Further courses built on it: -	

Analytical tools of descriptive geometry: analytical geometry of projections, oblique and orthogonal axonometry, central projection, central axonometry. Curves and surfaces. Hermite, Bézier curves and surfaces, B-splines. Representation of polyhedra.

## Literature

Recommended:

- M. K. Agoston. Computer Graphics and Geometric Modeling. Springer-Verlag London Limited, 2005 ISBN 978-1-85233-818-3

- G. Farin. Curves and surfaces for computer-aided geometric design. Morgan Kaufmann, 5th edition, 2002 ISBN 978-1-55860-737-8

#### Schedule:

 $I^{st}$  week Basics of computer graphics I.  $2^{nd}$  week Basics of computer graphics II.  $3^{rd}$  week Realization of affine transformations.  $4^{th}$  week Plotting functions of one variable.  $5^{th}$  week Plotting curves in the plane.  $6^{th}$  week Projections. 7<sup>th</sup> week

Representation of convex polyhedra.  $\delta^{th}$  week Representation of surfaces.  $9^{th}$  week Models of curves, Hermite curves.  $10^{th}$  week Models of curves, Bézier curves.  $11^{th}$  week Spline interpolation  $12^{th}$  week Models of surfaces, Hermite and Bézier surfaces  $13^{th}$  week B-spline surfaces  $14^{th}$  week Representation of fractals

# **Requirements:**

- for a signature

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence.

# - for a grade

During the semester there are two tests. The minimum requirement for a grade is to get 50% of the total score of the two tests. The grade is given according to the following table:

Score	Grade
0-49	fail (1)
50-62	pass (2)
63-74	satisfactory (3)
75-86	good (4)
87-100	excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

# Person responsible for course: Dr. Ábris Nagy, assistant professor, PhD

Lecturer: Dr. Ábris Nagy, assistant professor, PhD

<b>Title of course</b> : Linear programming <b>Code</b> : TTMBE0607	<b>ECTS Credit points: 3</b>
Type of teaching, contact hours	
- lecture: 2 hours/week	
- practice: -	
- laboratory: -	
Evaluation: exam	
Workload (estimated), divided into contact hours:	
- lecture: 28 hours	
- practice: -	
- laboratory: -	
- home assignment: -	
- preparation for the exam: 62 hours	
Total: 90 hours	
Year, semester: 2 <sup>nd</sup> year, 1st semester	
Its prerequisite(s): TTMBE0102	
Further courses built on it: -	

The subject of linear programming. The simplex algorithm, optimality, degenerateness, unbounded objective function. Cycling and its avoidance; the lexicographic method and the Bland rule. Duality theory: reflexivity, weak and strong duality theorems; the complementary slackness theorem. Sensitivity analysis. Geometric aspects of linear programming. Applications: Carathéodory theorem, Farkas lemma, von Neumann's minimax theorem. Linear programming tasks in models.

### Literature

Compulsory:

Recommended:

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.

- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

# Schedule:

1<sup>st</sup> week

Introduction: Constrained optimization problems; linear programs in standard form; examples.  $2^{nd}$  week

Terminology; the simplex method.

3<sup>rd</sup> week

Unboundedness and degeneracy. Cycling. 4<sup>th</sup> week Perturbation method and the Bland rule. The basic theorem of linear programming. 5<sup>th</sup> week Duality theory: reflexivity and the weak duality theorem. 6<sup>th</sup> week Duality theory: the strong duality theorem; complementary slackness; dual simplex method. 7<sup>th</sup> week Linear programs in matrix form. 8<sup>th</sup> week The primal and dual simplex methods in matrix form. 9<sup>th</sup> week The primal-dual method; sensitivity analysis. 10<sup>th</sup> week Convex geometry in linear programming. 11<sup>th</sup> week Linear programming in convex geometry: Carathéodory theorem and Farkas' lemma. 12<sup>th</sup> week Matrix games. 13<sup>th</sup> week Neumann's minimax theorem. 14<sup>th</sup> week Further examples and applications. **Requirements:** Attendance at lectures is recommended, but not compulsory. - for a grade The course ends in an **examination**. The minimum requirement for the examination is 50%. Based on the score of the exam the grade for the examination is given according to the following table: Score Grade  $0-4^{\circ}$ 50-63-)

0-49	fail (1)
50-62	pass (2)
63-76	satisfactory (3)
77-88	good (4)
89-100	excellent (5)

Person responsible for course: Dr. Fruzsina Mészáros, senior assistant professor, PhD

Lecturer: Dr. Fruzsina Mészáros, senior assistant professor, PhD

<b>Title of course</b> : Linear programming <b>Code</b> : TTMBG0607	ECTS Credit points: 2
Type of teaching, contact hours	
- lecture: -	
- practice: 2 hours/week	
- laboratory: -	
Evaluation: mid-semester grade	
Workload (estimated), divided into contact hours:	
- lecture: -	
- practice: 28 hours	
- laboratory: -	
- home assignment: -	
- preparation for the tests: 32 hours	
Total: 60 hours	
Year, semester: 2 <sup>nd</sup> year, 1st semester	
Its prerequisite(s): TTMBE0102	
Further courses built on it:-	
Topics of course	
The subject of linear programming. The simplex algorithm, optimality objective function. Cycling and its avoidance: the lexicographic	y, degenerateness, unbounded method and the Bland rule

objective function. Cycling and its avoidance; the lexicographic method and the Bland rule. Duality theory: reflexivity, weak and strong duality theorems; the complementary slackness theorem. Sensitivity analysis. Geometric aspects of linear programming. Applications: Carathéodory theorem, Farkas lemma, von Neumann's minimax theorem. Linear programming tasks in models.

## Literature

Compulsory:

Recommended:

- Vanderbei, R.: Linear Programming, Foundations and Extensions, Kluwer Academic Publishers, 1998.

- Bertsimas, D.; Tsitsiklis, J.: Introduction to Linear Optimization, Athena Scientific Series in Optimization and Neural Computation, 6, Athena Scientific, Belmont, 1997.

#### Schedule:

 $I^{st}$  week The graphical method.  $2^{nd}$  week Simplex algorithm via dictionaries.  $3^{rd}$  week

Simplex algorithm via dictionaries; discovering unboundedness and degeneracy. 4<sup>th</sup> week Simplex algorithm via dictionaries; perturbation and Bland's rule. 5<sup>th</sup> week Simplex algorithm via simplex tableau. 6<sup>th</sup> week Primal-dual problems; the dual simplex method. 7<sup>th</sup> week Sensitivity analysis. 8<sup>th</sup> week Using computers in linear programming. 9<sup>th</sup> week Using computers in linear programming. 10<sup>th</sup> week Using computers in linear programming. 11<sup>th</sup> week Using computers in linear programming. 12<sup>th</sup> week Matrix games. 13<sup>th</sup> week Modeling. 14<sup>th</sup> week Modeling.

# **Requirements:**

# - for a practical

Participation at **practice classes** is compulsory. A student must attend the practice classes and may not miss more than three times during the semester. In case a student does so, the subject will not be signed and the student must repeat the course. A student can't make up any practice with another group. Attendance at practice classes will be recorded by the practice leader. Being late is equivalent with an absence. In case of further absences, a medical certificate needs to be presented. Active participation is evaluated by the teacher in every class. If a student's behavior or conduct doesn't meet the requirements of active participation, the teacher may evaluate his/her participation as an absence because of the lack of active participation in class.

During the semester there are two tests: one test in the 7<sup>th</sup> week and the other test in the 14<sup>th</sup> week. The minimum requirement for the tests respectively is 50%. Based on the score of the tests, the grade for the tests is given according to the following table:

Grade
fail (1)
pass (2)
satisfactory (3)
good (4)
excellent (5)

If the score of any test is below 50, students can take a retake test in conformity with the EDUCATION AND EXAMINATION RULES AND REGULATIONS.

Person responsible for course: Dr. Fruzsina Mészáros, senior assistant professor, PhD

Lecturer: Dr. Fruzsina Mészáros, senior assistant professor, PhD

<b>Title of course</b> : Basics of mathematics <b>Code</b> : TTMBG0001	ECTS Credit points: 0	
Type of teaching, contact hours		
- lecture: -		
- practice: 1 hours/week		
- laboratory: -		
Evaluation: signature		
Workload (estimated), divided into contact hours:		
- lecture: -		
- practice: 14 hours		
- laboratory: -		
- home assignment: -		
- preparation for the exam: 14 hours		
Total: 28 hours		
Year, semester: 1 <sup>st</sup> year, 1 <sup>st</sup> semester		
Its prerequisite(s): -		
Further courses built on it: -		
Topics of course		
Algebraic transformations. Solution of different type equations inequality systems. Basic notions of trigonometry and coordinates and the system of the syst	s, equation systems, inequalities and ate geometry.	
Literature		
Compulsory:		
- Recommended:		
A. Bérczes and Á. Pintér: College Algebra. University of Deb	recen, 2013.	
R. D. Gustafson: College algebra and trigonometry. Pacific G	rove, Brooks/Cole, 1986.	
Schedule:		
I <sup>st</sup> week		
Algebraic transformations, identities, simplification of rational algebraic expressions. $2^{nd}$ week		
Simplification of irrational algebraic expressions, rationalization of denominator.		
3 <sup>rd</sup> week		
Parametric linear equations, equation systems.		
4 <sup>th</sup> week		
Quadratic equations, equation systems.		
5 <sup>th</sup> week		
Parametric quadratic equations.		
6 <sup>th</sup> week		
Sign of linear and quadratic expressions, inequalities, inequality systems (table of signs).		
7 <sup>th</sup> week		
Equations containing absolute value.		
δ <sup></sup> week		

Trigonometry: geometric interpretation of trigonometric functions and basic properties. *9*<sup>th</sup> week

Identities of sum and difference of angle and trigonometric identities.

10<sup>th</sup> week

Trigonometric equations, inequalities. Method of phase shift.

11<sup>th</sup> week

Coordinate geometry: lines and circles in a plane, intersectional exercises. Distance of points and of point and line.

12<sup>th</sup> week

Lines and circles in the plane, exercises concerning tangent line.

13<sup>th</sup> week

Exponential function and its inverse, the logarithm.

14<sup>th</sup> week

Exponential and logarithmic equations, inequality.

# **Requirements:**

- for a signature

Attendance of classes are compulsory with the possibility of missing at most three classes during the semester. In case of further absences, a medical certificate needs to be presented, otherwise the signature is denied.

The course is evaluated on the basis of two written tests during the semester. The signature is given if the student obtains at least 60 percent of the total points.

If a student fail to pass at first attempt, then a retake of the tests is possible.

- for a grade

There is no grading in this course. *-an offered grade:* There is no grading in this course.

Person responsible for course: Dr. Nóra Györkös-Varga, assistant professor, PhD

Lecturer: Dr. Nóra Györkös-Varga, assistant professor, PhD